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Analysis of Composite Corridors

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Abstract

This work began as an attempt to find and catalog the mean values and temperatures of a well-defined set of relatively simple common Go positions, extending a similar but smaller catalog in Table E.10, Appendix E of the book, "Mathematical Go[1]".

The major surprises of our present work include the following

- A position of chilled value *2 (previously unknown in Mathematical Go).
- A surprisingly "warm" position, whose temperature is routinely underestimated even by very strong Go players.
- More insights into decompositions. Some positions decompose as a beginner might naively hope; others don't. One set of those which don't provides a basis for an extension of the "multiple invasions" theorem in the Mathematical Go book. This appears in our Section 5. In the new set of positions, like the old, a potential future shortage of liberties of the invading group results in a surprisingly hot temperature at one well-defined but far-from-obvious point along the invading group's frontier.

It is hoped that these results may someday provide the basis for further new insights and generalizations.

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1 Introduction

Appendix E of the book "Mathematical Go[1]" contains a collection of small, relatively cool Go positions and their chilled values. Among the highlights of this Appendix is Figure E.10 and its surroundings (pages 196-201 in the English edition; pages 199-204 in the Japanese translation), which is an extensive tabulation of simple corridors located along an edge of the board. This paper extends those results. Our preliminary long-range goal was to evaluate all positions in which:

- All nodes on the first row are initially empty except for the endpoints, each of which may be occupied by an immortal stone of either color
- All nodes on the second row are either empty or occupied by immortal stones,
- Every empty node on the second row (a "gap") lies between 2nd-row nodes occupied by stones of different colors.

The present paper investigates all such positions with only one gap, as well as many (but not all) cases with two gaps. For each such position, we obtain the mean value and a temperature. For games of temperature one, we also obtain the value of the infinitesimal to which the game chills. Our analysis includes not only these initial positions, but all of their orthodox descendants as well as some plausible unorthodox descendants.

1.1 Definition

We define the following terms for Go positions used in this article.

corridor All the stones on the second row are alive. There are two types of corridors. A *blocked corridor* is closed at one side of the first row with the stone of the same color as the corridor. The opponent may invade from the other side. An *unblocked corridor* is open to invasion on both sides. The left position of Figure 1 is the *blocked corridor of length i* and the right position is the *unblocked corridor of length i*. The short lines coming out of the stones on the second row denote these stones are alive.

Figure 1: Example of Corridors

socket We define the two types of sockets. We call the point "a" of the left positions of Figure 2 a *full socket* (or just *socket*) and the point "b" of the right positions of Figure 2 a *half socket.*

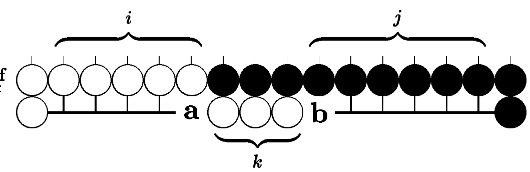
Figure 2: Tow types of sockets

dame An empty point which doesn't belong to the territory of either player. *Dames* are eventually filled at the end of the game.

2 Two Adjacent Corridors without Gaps

2.1 Blocked on Both Sides

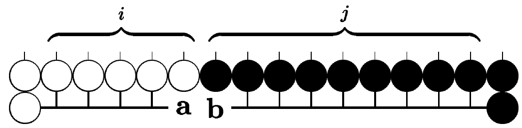
The following two tables show the mean value, temperature, ish-type, Black's play and White's play for each i, j and k of two adjacent blocked corridors without gaps[2]. The rows of the tables are sorted in lexicographic order of (j, i, k).

$a_{i,j,k} \stackrel{\text{def}}{=}$ 

i	j	k	Mean	Temp	ish	Black	White
0	≥ 0	≥ 1	$j - 2 + 2^{1-j}$	$1 - 2^{1-j}$		b	b
1	0	1	$\frac{1}{3}$	$\frac{1}{3}$		a	a
1	0	2	1	1	$\downarrow$	a	a
1	0	≥ 3	$k - 1\frac{1}{12}$	$k - 1\frac{1}{12}$		a	a
2	0	≥ 1	$k - 1$	k	$*$	a	a
≥ 3	0	1	$-i + 2\frac{1}{6}$	$1\frac{1}{6}$		a	a
≥ 3	0	≥ 2	$k - i + 1$	k		a	a
≥ 1	1	≥ 1	$-i + 1$	0			
1	≥ 2	1	$j - 1\frac{1}{3}$	$\frac{2}{3}$		b	b
1	≥ 2	2	$j - 1$	1	$\Downarrow *$	b	b
1	≥ 2	≥ 3	$j - 1$	1	$\vdash\!\dashv$	b	b
≥ 2	≥ 2	≥ 1	$j - i$	1	$\vdash\!\dashv$	b	b

Table 1: $a_{i,j,k}$: Blocked on Both Sides

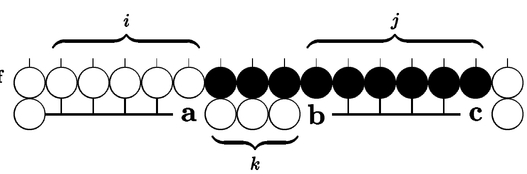
Table 2 is in the case of $k = 0$ and we assume $i \leq j$ without loss of generality.

$a_{i,j,0} \stackrel{\text{def}}{=}$ 

i	j	k	Mean	Temp	ish	Black	White
0	≥ 0	0	$j-2+2^{1-j}$	$1-2^{1-j}$		b	b
1	1	0	0	0			
1	2	0	$\frac{1}{2}$	$\frac{1}{2}$		b	b
≥ 2	≥ 2	0	$j-i$	1	$*$	a	b
1	≥ 3	0	$j-1\frac{2}{3}$	$\frac{2}{3}$		b	b

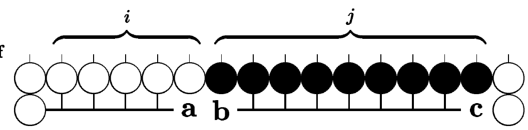
Table 2: $a_{i,j,0}$: Blocked on Both Sides

2.2 Blocked on Offensive Side and Unblocked on Defensive Side

$b_{i,j,k} \stackrel{\text{def}}{=}$ 

i	j	k	Mean	Temp	ish	Black	White
≥ 0	0	≥ 1	$-i$	0			
0	≥ 1	≥ 1	$j-4+2^{3-j}$	$1-2^{3-j}$		b,c	b,c
1	1	1	$-\frac{1}{3}$	$\frac{2}{3}$		b(=c)	b(=c)
1	1	2	0	1	$\Downarrow *$	b(=c)	b(=c)
1	1	≥ 3	0	1	$\vdash\!\!\dashv$	b(=c)	b(=c)
2	1	1	-1	1	$\downarrow$	b(=c)	b(=c)
≥ 2	1	≥ 2	$-i+1$	1	$\vdash\!\!\dashv$	b(=c)	b(=c)
≥ 3	1	1	$-i+1$	1	$\vdash\!\!\dashv$	b(=c)	b(=c)
1	2	≥ 1	0	0			
≥ 2	≥ 2	≥ 1	$j-i-2+2^{2-j}$	$1-2^{2-j}$		b	c
1	3	1	$\frac{1}{3}$	$\frac{1}{3}$		b	c
1	≥ 3	≥ 2	$j-3+2^{2-j}$	$1-2^{2-j}$		b	c
1	≥ 4	1	$j-3\frac{1}{3}+\frac{2^{4-j}}{3}$	$1-\frac{2^{4-j}}{3}$		c	c

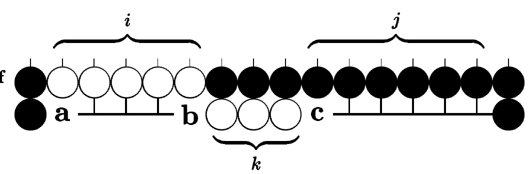
Table 3: $b_{i,j,k}$: Blocked on Offensive Side and Unblocked on Defensive Side

$b_{i,j,0} \stackrel{\text{def}}{=}$ 

i	j	k	Mean	Temp	ish	Black	White
≥ 0	0	0	$-i$	0			
0	≥ 1	0	$j-4+2^{3-j}$	$1-2^{3-j}$		b,c	b,c
≥ 0	1	0	$-i+1-2^{-i}$	$1-2^{-i}$		b(=c)	b(=c)
1	2	0	0	0			
≥ 2	≥ 2	0	$j-i-2+3\cdot 2^{1-j}$	$1-2^{1-j}$		a	a,b
1	3	0	0	0			
1	≥ 4	0	$j-3\frac{2}{3}+\frac{2^{4-j}}{3}$	$1-\frac{2^{4-j}}{3}$		c	c

Table 4: $b_{i,j,0}$: Blocked on Offensive Side and Unblocked on Defensive Side

2.3 Unblocked on Offensive Side and Blocked on Defensive Side

$c_{i,j,k} \stackrel{\text{def}}{=}$ 

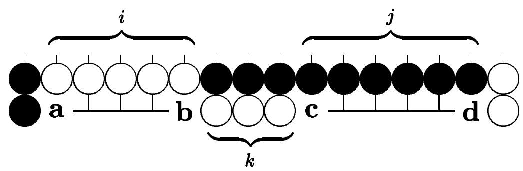
i	j	k	Mean	Temp	ish	Black	White
0	≥ 0	≥ 1	$j+2k$	0			
1	≥ 0	≥ 1	$k+j-1+2^{-j}$	$k+1-2^{-j}$		a(=b)	a(=b)
2	0	1	$\frac{2}{3}$	$\frac{2}{3}$		b	b
≥ 2	0	≥ 2	$k-i+3-3\cdot 2^{1-i}$	$k-2^{1-i}$		b	b
3	0	1	$\frac{1}{4}$	$\frac{3}{4}$	$*$	b	b
≥ 4	0	1	$-i+4\frac{1}{6}-\frac{11}{3}2^{1-i}$	$\frac{7}{6}-\frac{5}{3}2^{1-i}$		b	b
2	1	≥ 1	0	0			
≥ 3	1	≥ 1	$-i+3-2^{2-i}$	$1-2^{2-i}$		a	a
2	≥ 2	1	$j-1\frac{1}{6}$	$\frac{5}{6}$		c	c
≥ 2	≥ 2	≥ 2	$j-i+2-2^{2-i}$	1	$\vdash\!\dashv$	c	c
3	≥ 2	1	$j-1\frac{5}{8}$	$\frac{7}{8}$		c	c
4	≥ 2	1	$j-2\frac{13}{48}$	$\frac{47}{48}$		c	c
≥ 5	≥ 2	1	$j-i+2-2^{2-i}$	1	$\vdash\!\dashv$	c	c

Table 5: $c_{i,j,k}$: Unblocked on Offensive Side and Blocked on Defensive Side

We omit the table of $c_{i,j,0}$, since $c_{i,j,0}$ is $-b_{j,i,0}$ obvious from the definition.

2.4 Unblocked on Both Sides

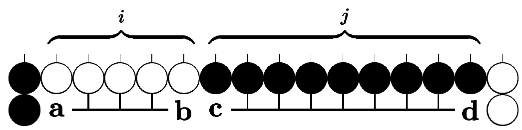
$d_{i,j,k} \stackrel{\text{def}}{=}$



i	j	k	Mean	Temp	ish	Black	White
≥ 0	i	≥ 1	0	0			
≥ 1	0	≥ 1	$-i+2-2^{1-i}$	$1-2^{1-i}$		a	a
≥ 3	$> i$	≥ 2	$j-i+2^{2-j}-2^{2-i}$	$1-2^{2-j}$		c	d
0	1	≥ 1	k	k		c(=d)	c(=d)
2	1	≥ 1	0	0			
≥ 3	1	≥ 1	$-i+3-2^{2-i}$	$1-2^{2-i}$		c(=d)	c(=d)s
0	≥ 2	1	j	1	$\downarrow +j.(\uparrow *)$	d	d
0	≥ 2	≥ 2	$2k+j-2$	1	$0^{j-2}\|$ ⧾	d	d
1	2	≥ 1	k	k		a(=b)	a(=b)
≥ 3	2	≥ 1	$-i+3-2^{2-i}$	$1-2^{2-i}$		a	a
$> j$	≥ 3	≥ 2	$j-i+2^{2-j}-2^{2-i}$	$1-2^{2-i}$		c	a
1	≥ 3	≥ 1	$k+j-3+2^{2-j}$	$k+1-2^{2-j}$		a(=b)	a(=b)
2	3	≥ 1	$\frac{1}{2}$	$\frac{1}{2}$		c	b(,c,d)
2	≥ 3	≥ 2	$j-3+2^{2-j}$	$1-2^{2-j}$		c	d
4	3	1	$-\frac{3}{4}$	$\frac{3}{4}$		a,c	a
≥ 5	≥ 3	1	$j-i+2^{2-j}-2^{2-i}$	$max\{1-2^{2-j}, 1-2^{2-i}\}$		c	a or d
2	4	1	$1\frac{1}{6}$	$\frac{2}{3}$		c	d
3	4	1	$\frac{3}{4}$	$\frac{3}{4}$		c	b,d
2	≥ 5	1	$j-3\frac{1}{6}+\frac{2^{4-j}}{3}$	$1-\frac{2^{4-j}}{3}$		d	d
3	5	1	$1\frac{1}{2}$	$\frac{1}{2}$		d	b,d
4	5	1	$\frac{7}{8}$	$\frac{7}{8}$		c	b,d
3	≥ 6	1	$j-3\frac{5}{8}+2^{2-j}$	$1-2^{2-j}$		d	d
4	6	1	$1\frac{13}{16}$	$\frac{15}{16}$		c	d
4	7	1	$2\frac{37}{48}$	$\frac{23}{24}$		c	d
4	≥ 8	1	$j-4\frac{7}{24}+2^{3-j}$	$1-2^{3-j}$		c	d

Table 6: $d_{i,j,k}$: Unblocked on Both Sides

In table 7, we assume $i \leq j$ without loss of generality.

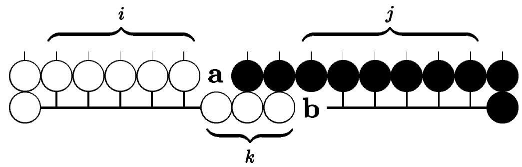
$d_{i,j,0} \stackrel{\text{def}}{=}$ 

i	j	k	Mean	Temp	ish	Black	White
≥ 0	i	0	0	0			
≥ 2	$> i$	0	$j - i - 3 \cdot 2^{1-i} + 2^{2-j}$	$1 - 2^{2-j}$		c	d
0	≥ 1	0	$j - 2 + 2^{1-j}$	$1 - 2^{1-j}$		d	d
1	2	0	0	0			
1	≥ 2	0	$j - 3 + 2^{2-j}$	$1 - 2^{2-j}$		a(=b),d	a(=b),d
2	3	0	0	0			
2	≥ 4	0	$j - 3\frac{1}{2} + 2^{2-j}$	$1 - 2^{2-j}$		d	d

Table 7: $d_{i,j,0}$: Unblocked on Both Sides

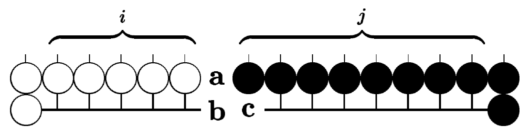
3 Two Adjacent Corridors with Gaps

3.1 Blocked on Both Sides

$A_{i,j,k} \stackrel{\text{def}}{=}$ 

i	j	k	Mean	Temp	ish	Black	White
0	≥ 0	≥ 1	$j-2+2^{1-j}$	$1-2^{1-j}$		b	b
1	0	1	$-\frac{1}{3}$	$\frac{2}{3}$		a	a
1	0	2	$1-i$	1	$\Downarrow *$	a	a
1	0	≥ 3	$1-i$	1	$\vdash\!\!\dashv$	a	a
2	0	1	$1-i$	1	$\downarrow$	a	a
2	0	≥ 2	$1-i$	1	$\vdash\!\!\dashv$	a	a
≥ 3	0	≥ 1	$1-i$	1	$\vdash\!\!\dashv$	a	a
≥ 1	1	≥ 1	$-i+\frac{1}{2}$	$\frac{1}{2}$		a	a
≥ 1	2	≥ 1	$1-i$	0		a,b	b
1	≥ 3	1	$j-2\frac{1}{6}$	$\frac{5}{6}$		a,b	b
1	≥ 3	2	$j-2$	1	$\Downarrow\!\!\!\downarrow$	a,b	b
1	≥ 3	≥ 3	$j-2$	1	$\vdash\!\!\dashv \mid 0$	a,b	b
2	≥ 3	1	$j-3$	1	$\Downarrow *$	a,b	b
≥ 2	≥ 3	≥ 2	$j-i-1$	1	$\vdash\!\!\dashv \mid 0$	a,b	b
2	≥ 3	≥ 2	$j-3$	1	$\vdash\!\!\dashv \mid 0$	a,b	b
≥ 3	≥ 3	≥ 1	$j-i-1$	1	$\vdash\!\!\dashv \mid 0$	a,b	b

Table 8: $A_{i,j,k}$: Blocked on Both Sides

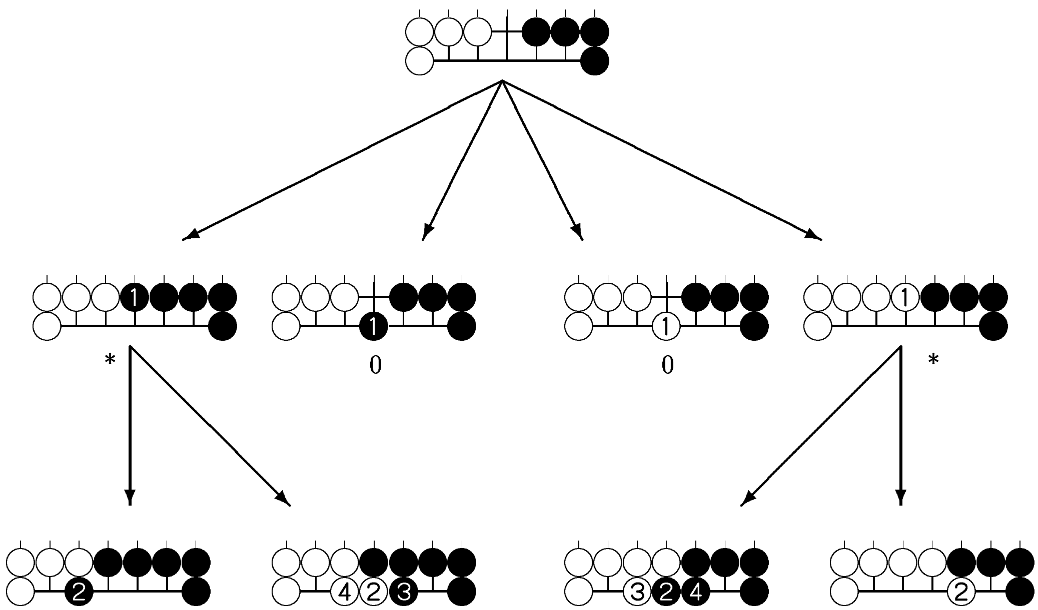
$A_{i,j,0} \stackrel{\text{def}}{=}$ 

i	j	k	Mean	Temp	ish	Black	White
0	≥ 0	0	$j-2+2^{1-j}$	$1-2^{1-j}$		b	b
≤ 1	≤ 1	0	0	0			
0	≥ 2	0	$j-2+\frac{1}{2}$	$\frac{1}{2}$		c	c
1	≥ 2	0	$j-1\frac{1}{3}$	$\frac{5}{6}$		b	b
2	2	0	0	1	$*2$	a,b	a,b
≥ 3	2	0	$2-i$	1	$\Uparrow *\mid 0,*$	b	a,b
2	≥ 3	0	$j-2$	1	$0,*\mid \Downarrow *$	a,b	b
≥ 3	≥ 3	0	$j-i$	1	$0\mid \boldsymbol{+} \,\|\, \boldsymbol{-}\mid 0$	b	b

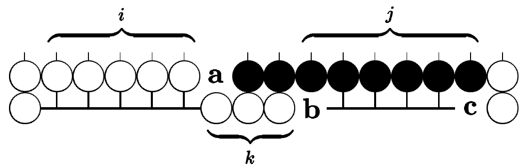
Table 9: $A_{i,j,0}$: Blocked on Both Sides

In table 9, we assume $i \leq j$ without loss of generality.

We found the new positions whose ish-types are previously unknown as Go positions. They are $*2$, $\{0,*\mid \Downarrow *\}$, $\{\Uparrow *\mid 0,*\}$ and $\{0\mid \boldsymbol{+} \,\|\, \boldsymbol{-}\mid 0\}$ listed at the bottom four rows of Table 9. The posision of $*2$ and its game tree are shown below.

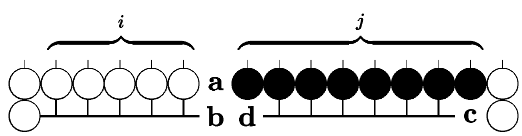


3.2 Blocked on Offensive Side and Unblocked on Defensive Side

$B_{i,j,k} \stackrel{\text{def}}{=}$ 

i	j	k	Mean	Temp	ish	Black	White
≥ 1	≤ 1	≥ 1	$-i$	0			
≥ 1	2	≥ 1	$-i+\frac{1}{2}$	$\frac{1}{2}$		a	a
0	≤ 3	≥ 1	0	0			
1	3	1	$-\frac{1}{3}$	$\frac{2}{3}$		a	a
1	≥ 3	≥ 2	$j-4+3\cdot 2^{1-j}$	$1-2^{1-j}$		a	a
≥ 2	≥ 3	≥ 1	$j-i-3+3\cdot 2^{1-j}$	$1-2^{1-j}$		a	a
0	≥ 4	≥ 1	$j-4+2^{3-j}$	$1-2^{3-j}$		b,c	b,c
1	4	1	$\frac{1}{4}$	$\frac{3}{4}$		a	a
1	5	1	1	$\frac{2}{3}$		c	c
1	≥ 6	1	$j-4\frac{1}{6}+\frac{1}{3}\cdot 2^{4-j}$	$1-\frac{1}{3}\cdot 2^{4-j}$		c	c

Table 10: $B_{i,j,k}$: Blocked on Offensive Side and Unblocked on Defensive Side

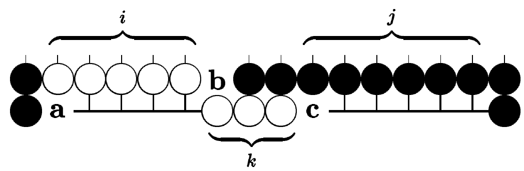
$B_{i,j,0} \stackrel{\text{def}}{=}$ 

i	j	k	Mean	Temp	ish	Black	White
≥ 0	0	0	$-i-2^{-1-i}$	$1-2^{-1-i}$		a	a
≥ 0	1	0	$-i+1-2^{-i}$	$1-2^{-i}$		a,c	b
0	$3\geq j\geq 1$	0	0	0			
0	2	0	0	0			
≥ 1	2	0	$-i+1$	0			
≥ 3	3	0	$-i+1\frac{1}{2}$	$\frac{1}{2}$		a,b	a
0	≥ 4	0	$j-3\frac{1}{2}+2^{2-j}$	$1-2^{2-j}$		c	c

Table 11: $B_{i,j,0}$: Blocked on Offensive Side and Unblocked on Defensive Side

3.3 Unblocked on Offensive Side and Blocked on Defensive Side

$C_{i,j,k} \stackrel{\text{def}}{=}$

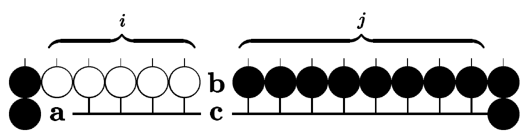


i	j	k	Mean	Temp	ish	Black	White
1	0	≥ 1	0	0			
≥ 2	0	≥ 1	$-i+3-2^{2-i}$	$1-2^{2-i}$		b	b
≥ 2	1	≥ 1	$-i+2\frac{1}{2}-2^{1-i}$	$1-2^{1-i}$		a	a
$1,2$	≥ 1	≥ 1	$j-2+2^{1-j}$	$1-2^{1-j}$		c	c
≥ 3	2	≥ 1	$-i+3-2^{1-i}$	$1-2^{1-i}$		a	a
≥ 2	≥ 3	≥ 1	$j-i+1+2^{3-i-j}-2^{2-i}$	$1-2^{3-i-j}$		c	c

Table 12: $C_{i,j,k}$: Unblocked on Offensive Side and Blocked on Defensive Side

A conjectured generalization of the last line of this table appears in Section 5.
We omit the table of $C_{i,j,0}$, since $C_{i,j,0}$ is $-B_{j,i,0}$ obvious from the definition.

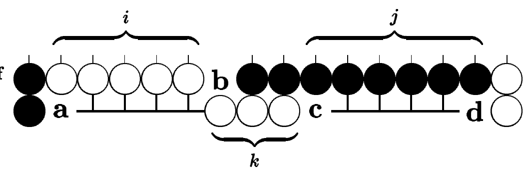
$C_{i,j,0} \stackrel{\text{def}}{=}$



i	j	k	Mean	Temp	ish	Black	White
0	≥ 0	0	$j+2^{-1-j}$	$1-2^{-1-j}$		b	b
1	≥ 0	0	$j-1+2^{-j}$	$1-2^{-j}$		a,b	c
$3 \geq i \geq 1$	0	0	0	0			
≥ 4	0	0	$-i+3\frac{1}{2}-2^{2-i}$	$1-2^{2-i}$		a	a
2	≥ 1	0	$j-1$	0			
3	≥ 3	0	$j-1\frac{1}{2}$	$\frac{1}{2}$		c	c

Table 13: $C_{i,j,0}$: Unblocked on Offensive Side and Blocked on Defensive Side

3.4 Unblocked on Both Sides

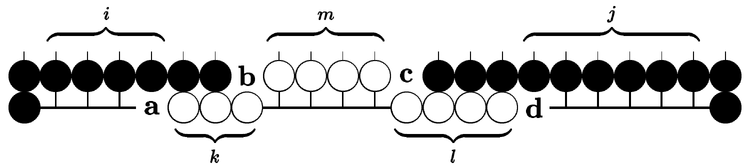
$D_{i,j,k} \stackrel{\text{def}}{=}$ 

i	j	k	Mean	Temp	ish	Black	White
≤ 2	≤ 2	0	0	0			
≥ 1	$< i$	≥ 2	$j - i - 1 + 3 \cdot 2^{1-j} - 2^{1-i}$	$1 - 2^{1-i}$		a	a
≥ 1	i	≥ 2	$j - i - 1 + 2^{3-j} - 2^{2-i}$	$1 - 2^{3-j}$		a,b,c	a,b
≥ 1	$i + 1$	≥ 2	$j - i - 1 + 2^{3-j} - 2^{2-i}$	$1 - 2^{3-j}$		b,c	b
≥ 1	$i + 2$	≥ 2	$j - i - 1 + 2^{3-j} - 2^{2-i}$	$1 - 2^{3-j}$		b,c	b,d
≥ 1	$\geq i + 3$	≥ 2	$j - i - 1 + 2^{3-j} - 2^{2-i}$	$1 - 2^{3-j}$		b,c	b

Table 14: $D_{i,j,k}$: Unblocked on Both Sides

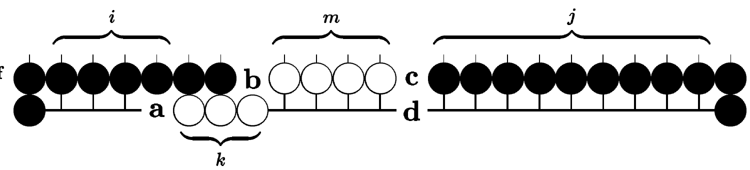
4 Three Adjacent Corridors with Gaps

4.1 Blocked on Both Outer Corridors

$X_{i,j,k,l,m} \stackrel{\text{def}}{=}$ 

i	j	k	l	m	Mean	Temp	ish	Black	White
0	0	1	1	2	$-\frac{1}{3}$	$\frac{2}{3}$		b,c	b,c
0	0	≥ 2	1	2	$-\frac{1}{4}$	$\frac{3}{4}$		c	b,c
1	0	≥ 1	1	2	$-\frac{5}{8}$	$\frac{7}{8}$		c	c
2	0	≥ 1	1	2	$-\frac{1}{8}$	$\frac{7}{8}$		c	c
3	0	1	1	2	$\frac{37}{48}$	$\frac{43}{48}$		a	a
≥ 3	0	≥ 2	1	2	$i - 2\frac{1}{4} + 2^{-1-i}$	$1 - 2^{-1-i}$		a	a
≥ 4	0	1	1	2	$i - 2\frac{1}{4}$	$\frac{11}{12}$		a	a
0	1	1	≥ 1	2	$-\frac{5}{8}$	$\frac{7}{8}$		b	b
0	1	≥ 2	≥ 1	2	$-\frac{1}{2}$	1		b	b
1	1	≥ 1	≥ 1	2	-1	0			
2	1	≥ 1	≥ 1	2	$-\frac{1}{2}$	$\frac{1}{2}$		c	c
≥ 3	1	1	≥ 1	2	$i - 2\frac{9}{16}$	$\frac{15}{16}$		a	a
≥ 3	1	≥ 2	≥ 1	2	$i - 2\frac{1}{2}$	1		a	a
0	2	≥ 2	1	2	0	1		a	a
1	2	≥ 2	1	2	$-\frac{1}{2}$	$\frac{1}{2}$		b	b
2	2	≥ 2	1	2	0	0			
2	2	≥ 2	≥ 2	2	0	0			
≥ 3	2	≥ 2	1	2	$i - 2$	1		a	a
0	≥ 3	≥ 2	1	2	$j - 2\frac{1}{8}$	$\frac{7}{8}$		d	d
1	≥ 3	≥ 2	1	2	$j - 2\frac{9}{16}$	$\frac{15}{16}$		d	d
2	≥ 3	≥ 2	1	2	$j - 2\frac{1}{16}$	$\frac{15}{16}$		d	d
≥ 3	≥ 3	≥ 2	1	2	$i + j - 4\frac{1}{8} + 2^{-2-i}$	$1 - 2^{-2-i}$		a	a
≥ 3	≥ 3	≥ 2	≥ 2	2	$i + j - 4$	1		a,b,c,d	a,d

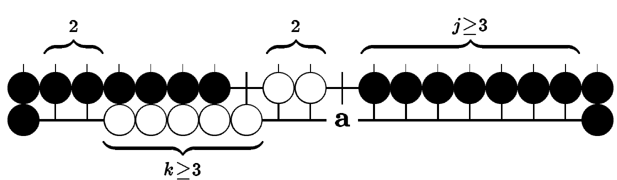
Table 15: $X_{i,j,k,l,m}$: Three Adjacent Corridors with Gap. Blocked on Both Outer Corridors

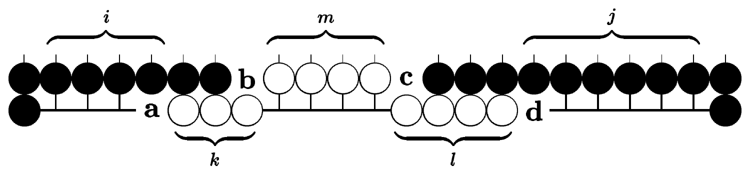
$X_{i,j,k,0,m} \stackrel{\text{def}}{=}$ 

i	j	k	l	m	Mean	Temp	ish	Black	White
0	0	0	0	2	0	0			
≥ 1	0	≥ 1	0	2	$i-2+2^{1-i}$	$1-2^{1-i}$		a	a
0	≤ 1	≥ 1	0	2	0	0			
≥ 1	≤ 1	0	0	2	$i-1$	0			
1	1	≥ 1	0	2	$-\frac{1}{3}$	$\frac{2}{3}$		d	d
2	1	≥ 1	0	2	$\frac{1}{4}$	$\frac{3}{4}$		d	d
≥ 3	1	≥ 1	0	2	$i-2+2^{-i}$	$1-2^{-i}$		a	a
$\geq j$	≥ 2	0	0	2	$i+j-2\frac{1}{3}$	$\frac{5}{6}$		a	a
0	≥ 2	1	0	2	$j-1\frac{1}{3}$	$\frac{2}{3}$		d	d
0	≥ 2	≥ 2	0	2	$j-1\frac{1}{4}$	$\frac{3}{4}$		d	d
1	≥ 2	≥ 1	0	2	$j-1\frac{5}{8}$	$\frac{7}{8}$		d	d
2	2	0	0	2	$1\frac{3}{4}$	$\frac{3}{4}$		a,d	a,d
2	≥ 2	$1,2$	0	2	$j-1\frac{1}{8}$	$\frac{7}{8}$		d	d
2	2	≥ 3	0	2	1	1		d	d
3	≥ 2	≥ 2	0	2	$j-\frac{1}{8}$	$\frac{7}{8}$		d	d
$3,4$	≥ 2	1	0	2	$i+j-3\frac{11}{48}$	$\frac{43}{48}$		a	a
≥ 4	≥ 2	≥ 2	0	2	$i+j-3\frac{1}{4}+2^{-i}$	$1-2^{-i}$		a	a
≥ 5	≥ 2	1	0	2	$i+j-3\frac{1}{4}$	$\frac{11}{12}$		a	a
2	≥ 3	≥ 3	0	2	$j-1\frac{1}{32}$	$1\frac{1}{32}$		d	d

Table 16: $X_{i,j,k,0,m}$: Three Adjacent Corridors with Gap. Blocked on Both Outer Corridors

In the bottom row of Table 16 we can find the temperature is greater than one. The figure below is an example of this position. It seems to be an ordinary position like any other, but its temperature is $1\frac{1}{32}$.



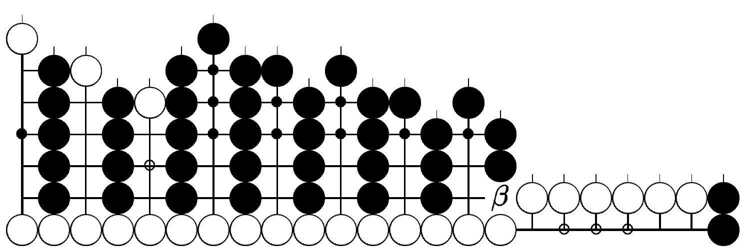
$X_{i,j,k,l,m} \stackrel{\text{def}}{=}$ 

i	j	k	l	m	Mean	Temp	ish	Black	White
0	0	1	≥ 1	3	-1	1	$\downarrow$	b,c	
0	0	≥ 2	≥ 2	3	-1	1	$\vdash\!\dashv$	b or c	b or c
1	0	≥ 1	≥ 1	3	$-1\frac{1}{2}$	1	$\vdash\!\dashv$	c	c
2	0	≥ 1	≥ 1	3	-1	1	$\vdash\!\dashv$	c	c
1	1	≥ 1	≥ 1	3	-2	0			
2	1	≥ 1	≥ 1	3	$-1\frac{1}{2}$	$\frac{1}{2}$		c	c
1	2	≥ 1	≥ 1	3	$-1\frac{1}{2}$	$\frac{1}{2}$		b	b
≥ 3	1	≥ 1	≥ 1	3	$i-3\frac{1}{2}$	1	$\vdash\!\dashv\,\vert 0$	a	a
2	2	≥ 1	≥ 1	3	-1	0			
≥ 3	2	≥ 1	≥ 1	3	$i-3$	1	$\vdash\!\dashv\,\vert 0$	a	a

Table 17: $X_{i,j,k,l,m}$: (continued)

5 Bicolored Multiple Corridors

We begin with the reference diagram on Page 27 of Berlekamp-Wolfe. It shows a position along the western edge of the board. We change the bottom three rows to the new kind of socket of *half socket.* The figure below is rotated 90 degrees counter-clockwise from the original one to save spaces.



The values (all marked and chilled) of b_i, u_i, and s are as on page 27.
As in the book, s is the shortest unblocked corridor, of length $\log_2 \frac{8}{s}$.
The new socket contains a node called β, whose value is taken as

1	if β is occupied by White
0	if β is occupied by Black
1/2	if β is empty

Below the half socket is a (new) white corridor, whose value is taken as σ, a negative number. The length of this corridor, measured by counting the sequence of empty nodes along the first column, is $\log_2 \frac{2}{|\sigma|}$.

Here is the conjectured value of the entire position, v:

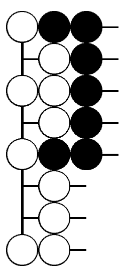
Case 1: IF $\beta + \sum b_i \geq 1$, then
$v = -1 + \beta + \sum b_i + \sigma + \sum u_i + s$

Case 2: IF $\beta + \sum b_i \leq 1$, and $2|\sigma| \leq s/2$, then
$v = \sum u_i + \sigma + s/2(1 + \beta + \sum b_i)$

Case 3: IF $\beta + \sum b_i \leq 1$, and $2|\sigma| \geq s/2$, then
$v = \sum u_i + s + |\sigma|(-3 + 2(\beta + \sum b_i))$

Case 4: IF $\beta + \sum b_i \leq 1$, and there is no s (and therefore no u_i) and $\beta = 0$ (full socket) then
$v = 0^n | \dagger$

COMMENTS: All this may be easier to understand if stated in terms of a rather different generalization of page 27, in which the bottom four rows are instead replaced with a sequence of k liberty-free sockets. Here is a bottom when $k = 3$:



Begin by considering the simplest case, in which each $b_i = 0$ or 1. Then $\sum b_i$ is the number of liberties enjoyed by the portion of the white string adjacent to the b's. Any liberties the white string has adjacent to the u's are ephemeral; they will disappear at a higher temperature. If each u_i is locally played canonically, then Black, if he is the first to move on that corridor, will play on the left side and take that liberty away. On the other hand, if White is the first to play on the corridor, then a black move at either ends reverses the value, so w.l.o.g, we can assume that as soon as White plays on any u_i, Black responds locally immediately to reverse the local u_i value and to take away that White liberty.

If there are k sockets, and if $\sum b_i \geq k$, then every u_i can be played canonically. However, if $\sum b_i < k$, (and each $b_i = 0$ or 1, and there are plenty of u_j's), then only the longest u_j's will decouple, each to its independent value, u_j. But Black will be able to play each of the $(k - \sum b_i)$ shortest u_j's in sente. When Black plays on one of them, say u, changing its value to $u/2$, White must respond immediately by filling a socket. So each of the shortest $(k - \sum b_i)$ u_j's has an effective value of $u_j/2$, while each of the longer u_j's has an effective value of u_j.

The more interesting case, of course, is when the b_i's are arbitrary. If $\sum b_i$ is an integer, then by canonical play within the b_i, we have mia, so that in the final result the $\sum b_i$ will be unchanged. The White string will then enjoy $\sum b_i$ liberties. And, as we have seen before, this implies that the requisite number of the shortest u's will have their values divided by 2, because when a canonical Black eventually plays there, White will be compelled to respond by filling a socket. But when a canonical Black plays first on a longer u_j, a canonical White will respond on the opposite end of the same corridor.

Finally, if $\sum b_i$ lies between two integers, then there is an issue of whether the play will round it up or down. The number of u_j's whose values need to be halved depends on the outcome. There will then be one critical u_j, called s. The values of longer u_j's will be unaffected, and the values of all shorter u_j's will be halved. The values of the b_i's interact with the value of the critical u_j, resulting in a term

$s/2((\sum b_i - \text{greatest integer in} \sum b_i) + 1)$

The play of the b_i's will determine whether this term becomes s or $s/2$.

All of this is proved in the book in the special case in which $k = 1$. I think the case of multiple k's was evaded because further complications arise if there are corridors emanating from the portions of White strings between different sockets. A typical Go player will consider such generalizations to be increasingly remote from anything he has directly encountered over the board.

If the white corridor at the bottom is the critical corridor, then the question to be resolved by playing the b's is whether its value is σ or 2σ. When both this white corridor and unblocked black corridors exist, then the white corridor must compete with the unblocked black corridors to determine which is the critical one. The conjecture stated above asserts that which corridor is critical is determined by a comparison of lengths. I don't yet have a plausible intuitive explanation for this; before examining the evidence of several specific cases, I might have been inclined to compare the values of the white and black corridors rather than their lengths.

The half socket appears in earlier sections of this paper. Such positions look very realistic. They readily include the new σ plus one other corridor. When there is an empty space (β) on the second column, someone may play on the first column opposite it. In some circumstances, this yields the last line of Table 12, which is a special case of our conjecture.

In most respects, the new β behaves like a blocked corridor of length two. But when $-\log_2|\sigma|$ is large, the half-point black mark associated with β belongs superimposed on a white mark immediately to its southwest. It is easiest to assume that β is not played until every other b_i has become 0 or 1. That turns out to be one canonical line. When there is exactly one other $b_i = 1/2$, and no u's or s, but σ is present, the position has value σ. If Black makes two consecutive plays, on b_i and on β, and White then fills, the new value is 2σ, which is the same as Black could have obtained by extending the bottom invasion instead of attacking White prematurely. But if Black does attack, the strictly dominant order in which to play them is to begin by playing the b_i to 0 rather than by playing the β to 0.

We have found many real pro endgames in which two adjacent regions are not-quite independent. The most common reason is that there is some potential pressure against the group which separates these two regions. Although each move will fall into one region or the other, plays which affect eyes or liberties of the potentially-pressured group may have a higher temperature than they would if the regions were truly independent. THIS SITUATION IS VERY COMMON. We now entertain visions of "Federation" software to help analyze it. I also think there is real hope of more theory to be discovered that might prove relevant to this problem.

I see our historical work on "multiple corridors", and the extension(s) discussed above, as a significant stepping stone in this direction. Although its direct applicability is too rare to excite very many Go players, I continue to be somewhat awed by its precision. It will certainly serve as a benchmark against which we can test any new theory or heuristics for evaluating a federation of two regions.

References

[1] Elwyn Berlekamp, John H. Conway and Rechard K. Buy: Winning Ways –for your Mathematical Plays–, Academic Press, New York, (1982).

[2] Elwyn Berlekamp and Dabid Wolfe: Mathematical Go –Chilling Gets the Last Point–, A.K.Peters, (1994).