On Counting Liberties in Capturing Races of Go

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Introduction

Applications of CGT to the Game of Go

- Endgame

  Berlekamp, Wolfe : "Mathematical Go", (1994)

  - Score is the sum of the number of points in territories and the number of prisoners, in Japanese rule.
  - But we don't need to assign the final score explicitly.
  - The score comes out of CGT itself by forbidding passes.
  - Cooling by one degree, or *chilling*, makes us get the last point, *tedomari*.

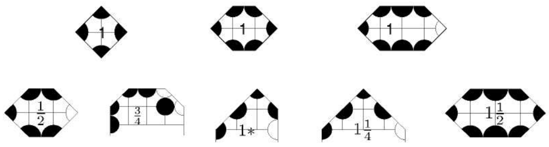
Introduction

Applications of CGT to the Game of Go

- Eyespace Value

  Landman : "Eyespace Values in Go", (1996)

  - **Bargo** : Eyespace Counting Game
    - The final score is the number of distinct eyes.
    - It doesn't matter how many points are contained in eye regions.
    - Chilling also works.



Introduction

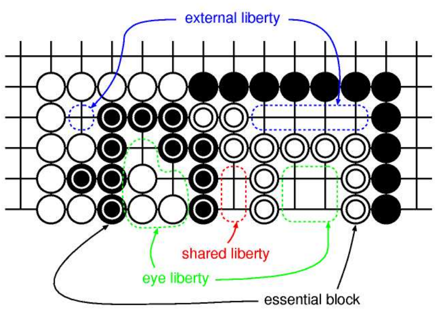
New Genre of Application of CGT to the Game of Go

- Counting Liberties in Capturing Races

  Nakamura (2003) :
  "Counting Liberties in Capturing Races using Combinatorial Game Theory" (in Japanese), IPSJ SIG-GI Technical Report, 2003 GI-9-5

- CGT can be applied to any situations that involve counting.
- Capturing Race, *Semeai* is a liberty counting game and is decomposable,
  - because for a semeai involving a big dragon we usually count liberties for each part of the dragon and sum up them.

Terminology for Semeai



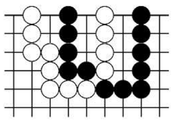
- **Plain liberty**
  - Liberty of essential block which is also adjacent to the opponent's safe or essential block.
  - Attacker can fill the plain liberty without making additional approach moves.

Related Research of Capturing Race

- Mueller : "Race to capture: Analyzing semeai in Go", GPW '99, (1999)
  - Classify capturing races into nine classes according to:
    - Type of external/shared liberties
    - Eye status

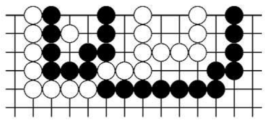
- class 0

Exactly one essential block of each color, only plain external and plain shared liberties, no eyes.



- class 1

One essential block of each color, may have one plain eye potentially containing one opponent nonessential block in a shape.



Mueller's Semeai Formula

- Evaluation formula for class 0 and class 1 semeai.

Δ : Advantage of external liberties for attacker
(difference of the number of liberties for each player's essential blocks)

S : Number of shared liberties

$$F = \begin{cases} S & (S = 0 \text{ or defender has an eye}) \\ S - 1 & (S > 0 \text{ and defender has no eye}) \end{cases}$$

Attacker can win the semeai, if $\Delta \geq F$ ....(1)

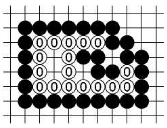
- This formula works well, but can be applied only for the case that all of the liberties are just simple numbers.
- In some cases, the number of liberties is not just a number, but a game whose value changes by each player's move.

Counting Liberties using CGT

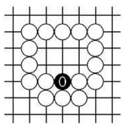
- Liberty Counting Game
  - Score is the number of liberties of essential blocks
  - Black's liberties are positive and White's are negative

$$G = \{G^L \mid G^R\}$$

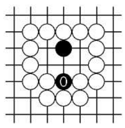
- Examples:



−4



{ 3 | 0 }



{ 4 | 0 }

- It seems easy, but in fact, we have to resolve a subtle problem to get these descriptions.

How should we assign the terminal score?

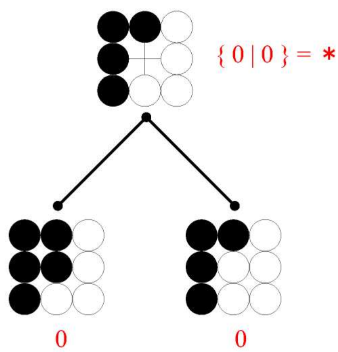
- In case of Go endgame,
  - We don't need to assign the terminal score explicitly.
  - The score comes out of CGT itself by forbidding passes, because the suicidal move is illegal.
- In case of eyespace value,
  - We have to assign the terminal score as the number of distinct eyes.
  - Once defender makes a secure eye space, attacker cannot destroy it.

How should we assign the terminal score?

- But in case of counting liberties,
  - Attacker can always fill the defender's liberties one by one.
  - There are no secure liberties in semeai.
    - Even eye liberties are not secure.
  - Defender should not play a move into his own liberty region, if he cannot extend the liberty at least one.
  - It is always bad move to take away his own liberty!
- We have to prune the defender's bad move explicitly in contrast to endgame, in which pruning is performed implicitly by means of legality.

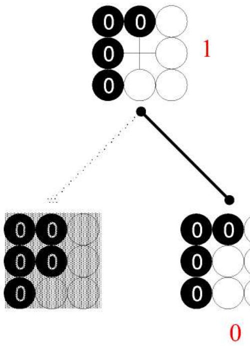
How should we assign the terminal score?

- Stone putting game with just one empty point



How should we assign the terminal score?

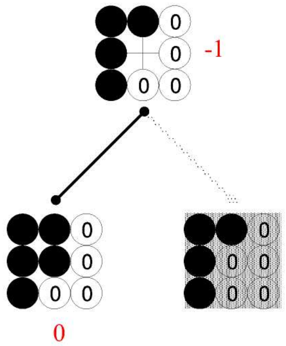
- Liberty counting game (Black's essential block)



Pruning bad move which reduces his own liberty

How should we assign the terminal score?

- Liberty counting game (White's essential block)



Pruning bad move which reduces his own liberty

Process for Assigning Terminal Liberty Score

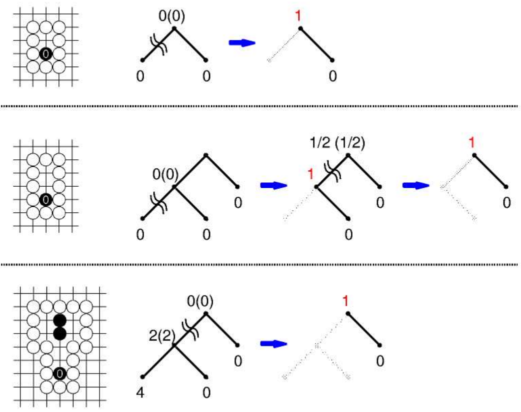
- Play out all legal moves for both players until the essential block has no liberty
  - At the present, it doesn't care whether the move is good or bad.
  - All the leaf nodes are zero.
  - In practice, however, we can count the number of liberties when all of liberties become plain liberty.
- From bottom to top, execute the following operations
  - If the temperature of a node is less than or equal to 1, prune the defender's branch.
  - If the resulting node becomes either of the following, replace it.

    $\{\ \mid n\}\ (n >= 0) \Longrightarrow n+1$

    $\{-n \mid\ \}\ (n >= 0) \Longrightarrow -n-1$

    (This replacement operation is exactly reverse manner contrary to conventional CGT.)

Process for Assigning Terminal Score



Semeai Game

- Sum of liberty counting games
- Assumed that each summand has just one essential block.
- Each summand is preprocessed using the process for assigning terminal score mentioned just before.
- Rules for play are same as Go. Suicidal move is forbidden.
- Player who fills all of the liberties of opponent's all essential blocks in all summands is a winner.
- In case of endgame and eyespace value, the smallest incentive of move is almost zero, *0-ish* and chilling works well.
- In case of semeai, the smallest incentive is just *1-ish*, because the attacker can always fill the opponent's liberties one by one.
- So, cooling by 2 degree must work to analyze semeai games.

Evaluation Method of Semeai Game

Supposed that G is a semeai game and g is Cool(G, 2).

- Case 1: g is an integer :
  - If g > 0, Black wins.
  - If g < 0, White wins.
  - If g = 0, first player wins.
- Case 2: n+1 > g > n (for some integer n) :
  - If Black plays first, he can round g up to n+1 in keeping his turn.
  - If White plays first, he can round g down to n in keeping his turn.
  - Check the resulting adjustment value using the conditions of case 1.
- Case 3: g <> n :
  - If Black plays first, he can round g up to n+1 in keeping his turn.
  - If White plays first, he can round g down to n-1in keeping his turn.
  - Check the resulting adjustment value using the conditions of case 1.

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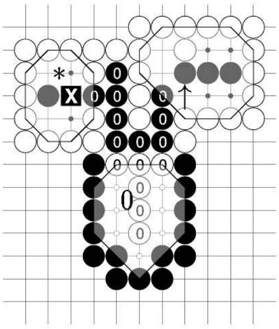
Problem 1



$\{4 \mid 0\} \overset{\text{cool by 2}}{\Longrightarrow} 2*$

$\{6 \mid \{4 \mid 0\}\} \overset{\text{cool by 2}}{\Longrightarrow} 4\uparrow$

-7

$\text{Total} = -1\uparrow* \; <> \; -1$

- Black's adjustment value is 0.
- Black can win, if he plays first.

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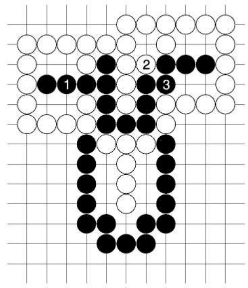
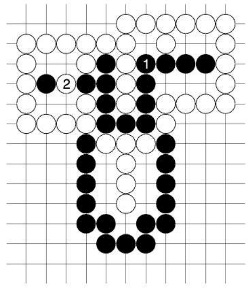
Problem 1

- Success



Black gets *tedomari*

- Failure



White gets *tedomari*

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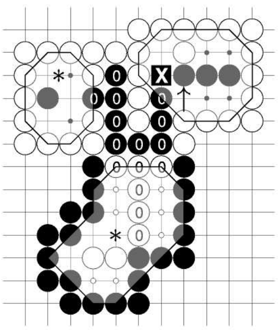
Problem 2



$\{4 \mid 0\} \overset{\text{cool by 2}}{\Longrightarrow} 2*$

$\{6 \mid \{4 \mid 0\}\} \overset{\text{cool by 2}}{\Longrightarrow} 4\uparrow$

$\{-5 \mid -9\} \overset{\text{cool by 2}}{\Longrightarrow} -7*$

$\text{Total} = -1\uparrow \; > \; -1$

- Black's adjustment value is 0.
- Black can win, if he plays first.

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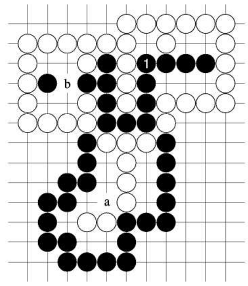
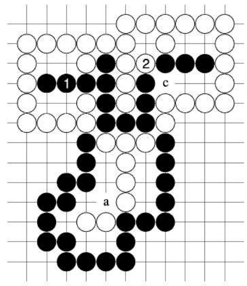
Problem 2

- Success



(a and b are miai)
Black gets tedomari

- Failure



(a and c are miai)
White gets tedomari

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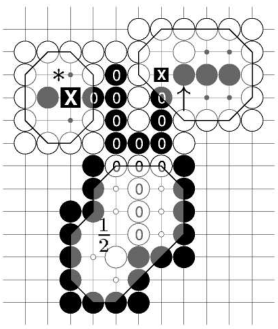
Problem 3



$\{4 \mid 0\} \overset{\text{cool by 2}}{\Longrightarrow} 2*$

$\{6 \mid \{4 \mid 0\}\} \overset{\text{cool by 2}}{\Longrightarrow} 4\uparrow$

$\{-5 \mid -8\} \overset{\text{cool by 2}}{\Longrightarrow} -7 + \frac{1}{2}$

$\text{Total} = -\frac{1}{2}\uparrow* \; > \; -1$

- Black's adjustment value is 0.
- Black can win, if he plays first.

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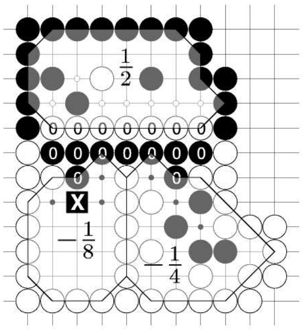
Problem 4



$\{-5 \mid -8\} \overset{\text{cool by 2}}{\Longrightarrow} -7 + \frac{1}{2}$

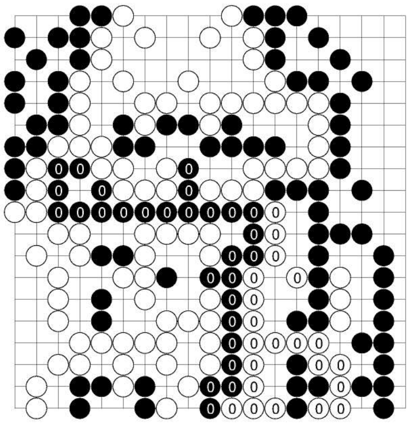
$\{\{\{8 \mid 5\} \mid 3\} \mid 1\} \overset{\text{cool by 2}}{\Longrightarrow} 3 - \frac{1}{8}$

$\{\{6 \mid 3\} \mid 1\} \overset{\text{cool by 2}}{\Longrightarrow} 3 - \frac{1}{4}$

$\text{Total} = -\frac{7}{8} \; > \; -1$

- Black's adjustment value is 0.
- Black can win, if he plays first.

Problem 5 (whole board problem)



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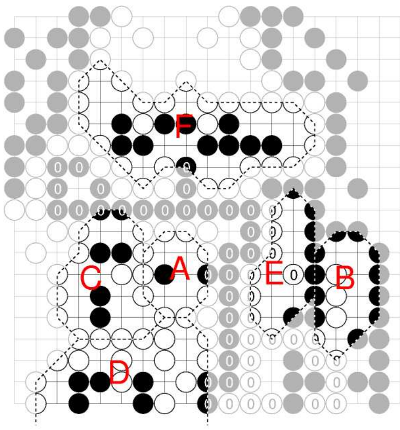
Problem 5



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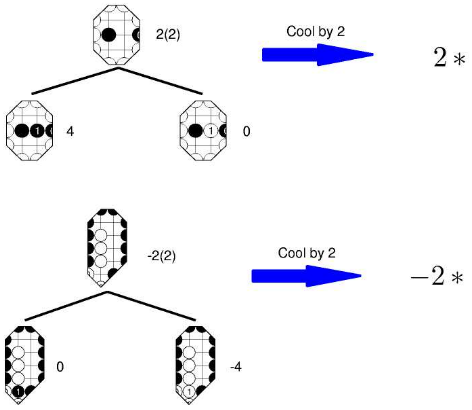
Problem 5 : A, B



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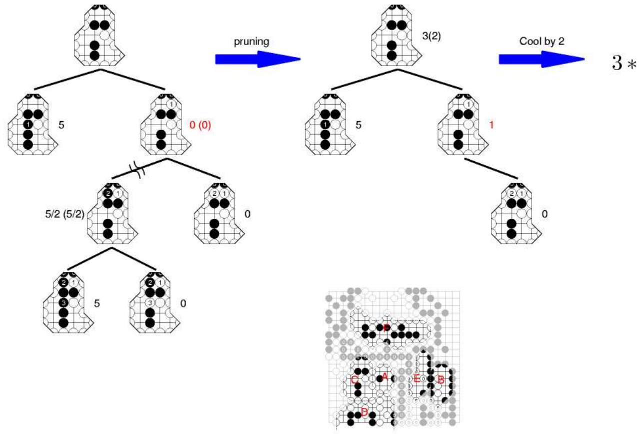
Problem 5 : C



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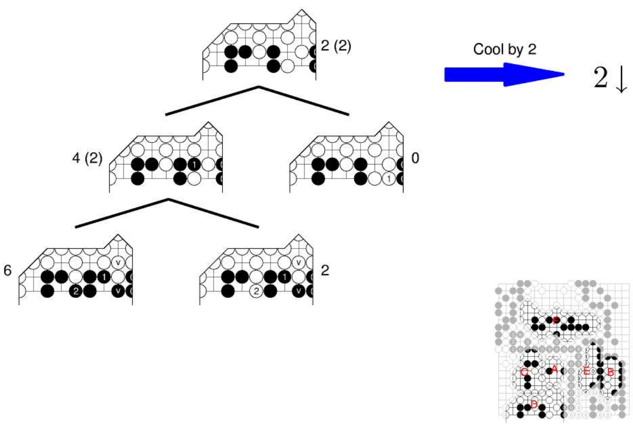
Problem 5 : D



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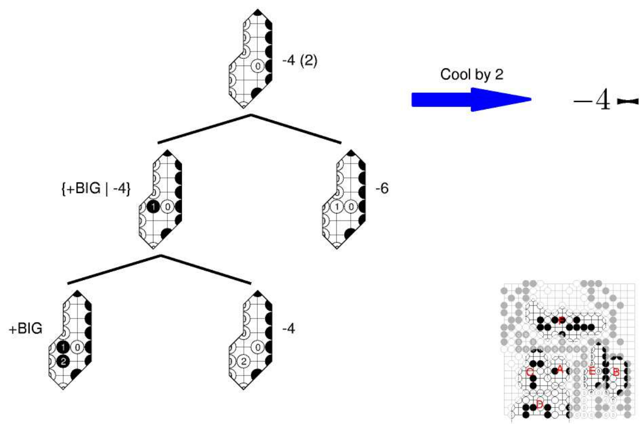
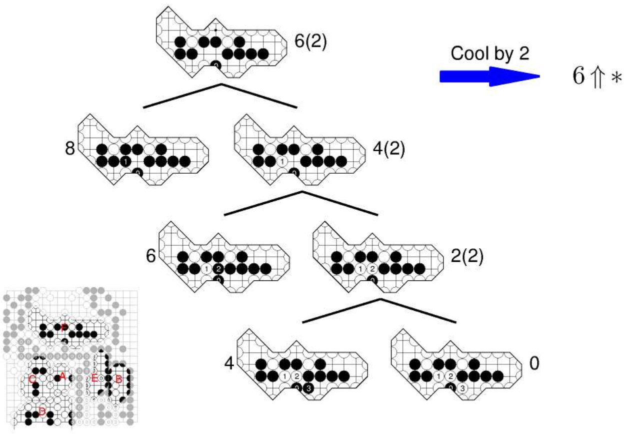
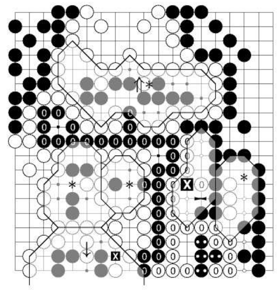
Problem 5 : E



Problem 5 : F



Problem 5



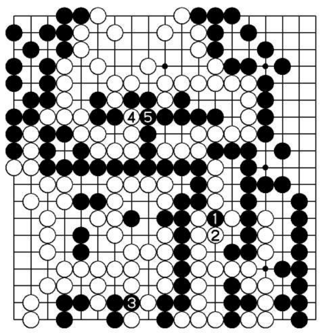
$$\text{Total} = (2*) + (-2*) + (3*) + (2\downarrow) + (-4\,\text{⧿}) + (6\Uparrow *) + 2 - 10 = -1\uparrow\text{⧿}$$

$$\text{Range:}\quad 0 > -1\uparrow\text{⧿} > -1$$

- Black's adjustment value is 0 ===> Black can win

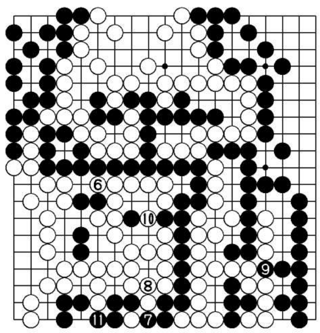
Correct sequence (1)

- B1 (throw in) at first, then B3.
- B5 in response to W4 is important.



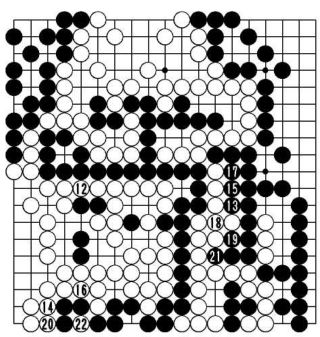
Correct sequence (2)

- All of W6, B9, W10 and B11 are same.
- Actual battle ends here and Black wins the semeai.



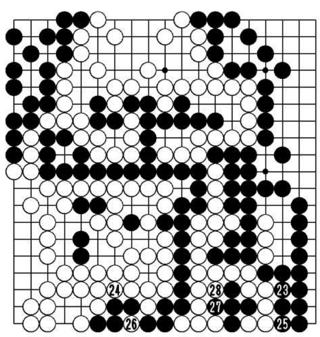
Correct sequence (3)

- Each player fills the liberties one by one from here.



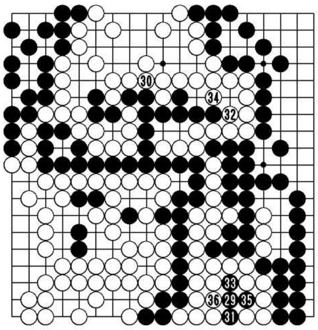
Correct sequence (4)

- Each player fills the liberties one by one.



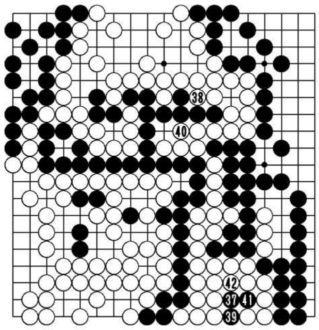
Correct sequence (5)

- Each player fills the liberties one by one.



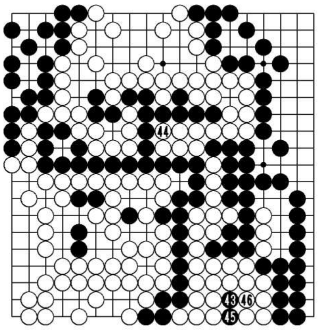
Correct sequence (6)

- Each player fills the liberties one by one.



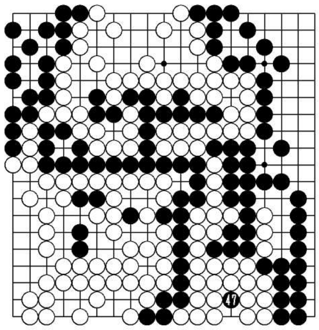
Correct sequence (7)

- Each player fills the liberties one by one.



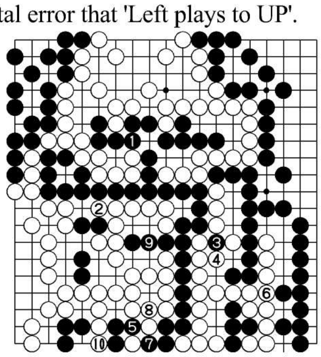
Correct sequence (8)

- 49 moves are required to capture the White's dragon eventually.

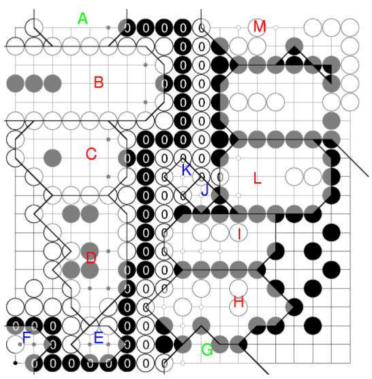


Failure

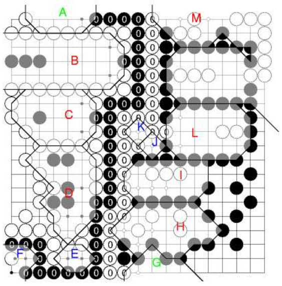
- If Black plays B1, White plays W2 and Black cannot win the semeai even if Black plays B3 and B5.
- B1 is a fatal error that 'Left plays to UP'.



Problem 6 (another whole board problem)



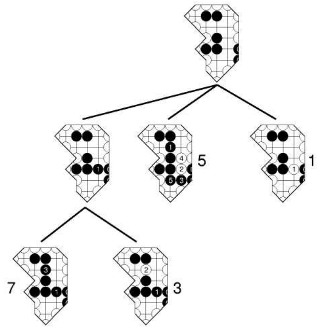
Problem 6



	value	atmic weight
A	1	0
B	$2\{⧿_4 \mid 0^3\}$	-3
C	$2\{⧿_1 \mid 0\}$	-1
D	$3{\uparrow}{*}$	1
E	2	0
F	3	0
G	-1	0
H	$-3{\uparrow}$	1
I	$-3{*}$	0
J	-1	0
K	-1	0
L	$-2\{0 \mid ⧾_2\}$	1
M	$-2\{0^2 \mid ⧾_4\}$	2
Total	0-ish	1

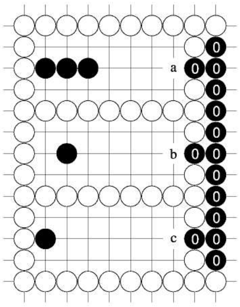
- Black can win by playing to B.

Problem 6

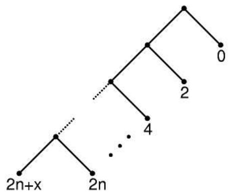


$\{\{7 \mid 3\}, 5 \mid 1\} \overset{\text{cool by 2}}{\Longrightarrow} 3\ \{0, * \mid 0\}$

Liberty Region of Corridor of Width 3



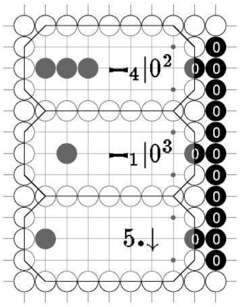
- Game tree



- corridor a n = 3, x = 8
- corridor b n = 4, x = 5
- corridor c n = 5, x = 4

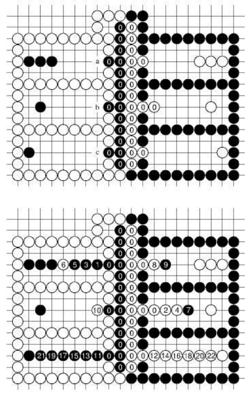
- What is the largest move?

Cooled Value of Each Corridor



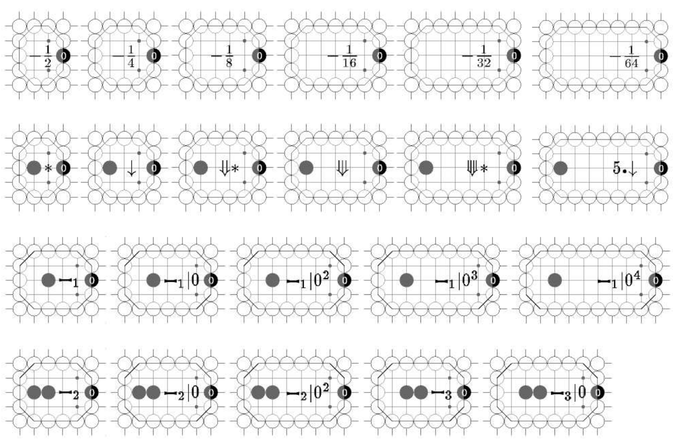
b > a > c

- Differencial Semeai Game



Black loses

Catalogue of Corridors



Evaluation of Semeai with Shared Liberties

G : Entire semeai game with the exclusion of shared libertiy regions

Δ' : Adjustment value of $Cool(G, 2)$ for attacker

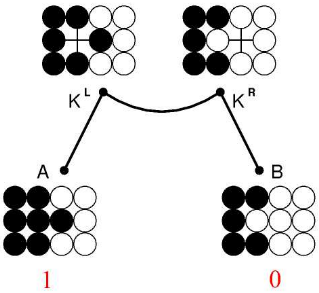
S : number of shared liberties

$$F = \begin{cases} S & (S = 0 \text{ or defender has an eye}) \\ S - 1 & (S > 0 \text{ and defender has no eye}) \end{cases}$$

$$\boxed{\text{Attacker can win the semeai, if } \Delta' \geq F} \ldots (2)$$

Ko

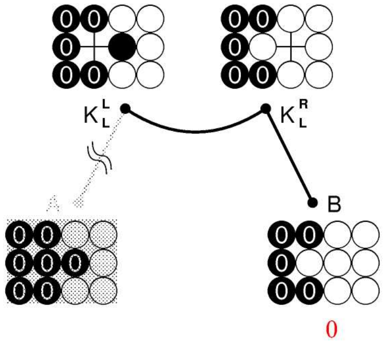
- Superscript of L and R means that which player has taken ko
- Game tree of half point ko



score as go endgame (supposed that K^R is the baseline)

Half Point Ko in Semeai

- Subscript L shows the player of the essential block



score as semeai : number of liberties of the essential block

Komaster Analysis

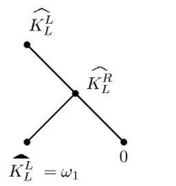
K^L_L K^R_L

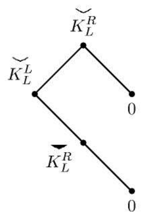
$2 \leqq \widehat{K}^L_L = \{\omega_1 \mid 0\} + 1$ $\widecheck{K}^L_L = 2$

$\widehat{K}^R_L = \{\omega_1 \mid 0\}$ $\widecheck{K}^R_L = \{2 \mid 0\} \Longrightarrow 1$

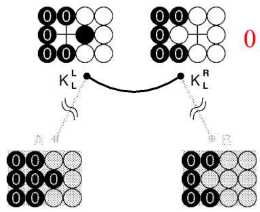
0

- Black komaster
- White komaster





Ko Fight in Semeai



- Attacker can continue to fight the whole semeai without filling the local ko

  ====> komonster

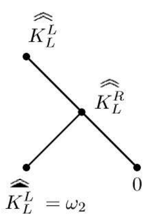
Komonster Analysis

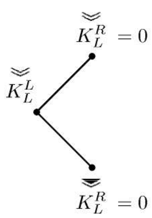
K^L_L K^R_L

$2 \leqq \widehat{\widehat{K}}^L_L = \{\omega_2 \mid 0\} + 1$ $\widecheck{\widecheck{K}}^L_L = 1$

$\widehat{\widehat{K}}^R_L = \{\omega_2 \mid 0\}$ $\widecheck{\widecheck{K}}^R_L = 0$

- Black komonster
- White komonster





Evaluating Semeai Including Kos

- Defender can be neither komaster nor komonster.
- If there are many kos, attacker can be komonster for only one ko and komaster for the other kos.
- The value of kos

$\widecheck{K}^L_L = 2$ $\widecheck{K}^R_L = 1$ $\widecheck{\widecheck{K}}^L_L = 1$ $\widecheck{\widecheck{K}}^R_L = 0$

$\widehat{K}^R_R = -2$ $\widehat{K}^L_R = -1$ $\widehat{\widehat{K}}^R_R = -1$ $\widehat{\widehat{K}}^L_R = 0$

  - superscript: player who has taken the ko
  - subscript: player of essential block

Evaluating Semeai Including Kos

- Decision table for the entire semeai game G including kos
- Each entry is just a concatenation of two results that Left plays first and Right plays first.

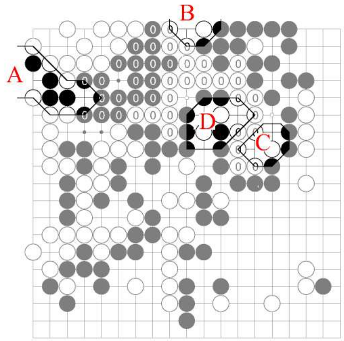
# of K_L	# of K_R	Value of G						
		3	2	1	0	-1	-2	-3
≥ 1	0	LL	LL	LL	Lk^0	$k^0k_L^1$	$k_L^1k_L^2$	$k_L^2k_L^3$
0	≥ 1	$k_R^3k_R^2$	$k_R^2k_R^1$	$k_R^1k^0$	k^0R	RR	RR	RR
≥ 1	≥ 1	$k_R^3k_R^2$	$k_R^2k_R^1$	$k_R^1k^0$	k^0k^0	$k^0k_L^1$	$k_L^1k_L^2$	$k_L^2k_L^3$

L : Left wins, R : Right wins

k_L^i, k_R^i : i-move approach ko for Left/Right, k^0 : real ko (hon-ko)

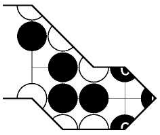
Example of Analysis using Real Game

- 11th Ryusei tournament in 2002
  - B: Hideki Komatsu (9p), W: Kunio Kamimura (9p)

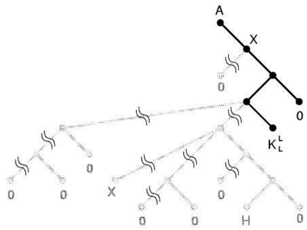


Analysis of Summand A

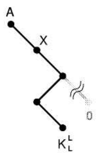
- Position A



- Game tree (pruned)



- Resulting game tree

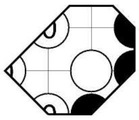


- Value of A

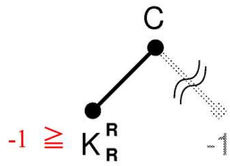
$$A = K_L^L + 2$$

Analysis of Summand C

- Position C



- Game tree (pruned)

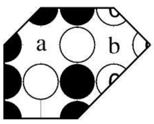


- Value of C

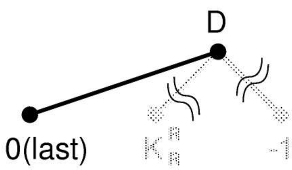
$$C = K_R^R - 1$$

Analysis of Summand D

- Position D



- Game tree (pruned)



- Value of D

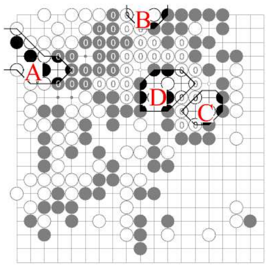
$$D = -1$$

(with the proviso of the last move after all the other liberties are filled)

Analysis of the Real Game

- Value of each summand

$$A = K_L^L + 2$$
$$B = K_R^R$$
$$C = K_R^R - 1$$
$$D = -1$$



- Total value

$$\begin{aligned} G &= A + B + C + D + 3 - 2 \\ &= (K_L^L + 2) + K_R^R + (K_R^R - 1) - 1 + 3 - 2 \\ &= K_L^L + 2 \cdot K_R^R + 1 \end{aligned}$$

Analysis of the Real Game

- Defender can be neither komaster nor komonster.
- If there are many kos, attacker can be komonster for only one ko and komaster for the other kos.

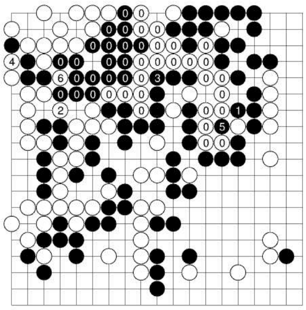
$$\breve{K}^L_L = 2 \quad \breve{K}^R_L = 1 \quad \breve{\breve{K}}^L_L = 1 \quad \breve{\breve{K}}^R_L = 0$$

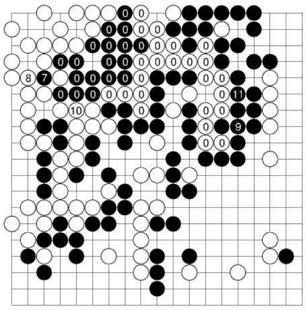
$$\widehat{K}^R_R = -2 \quad \widehat{K}^L_R = -1 \quad \widehat{\widehat{K}}^R_R = -1 \quad \widehat{\widehat{K}}^L_R = 0$$

$$\begin{aligned} G &= K^L_L + K^R_R + K^R_R + 1 \\ &= \breve{\breve{K}}^L_L + \widehat{\widehat{K}}^R_R + \widehat{K}^R_R + 1 \\ &= 1 + (-1) + (-2) + 1 \\ &= \underline{-1} \end{aligned}$$

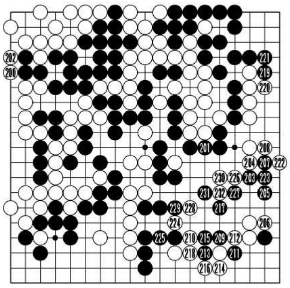
one move approach ko for Black

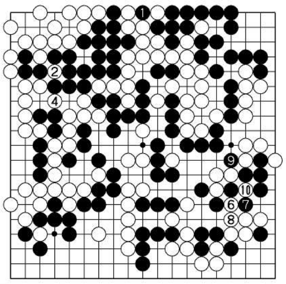
Analysis of the Real Game





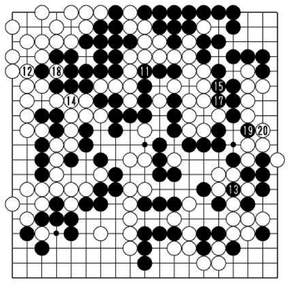
Sequence of the Real Game



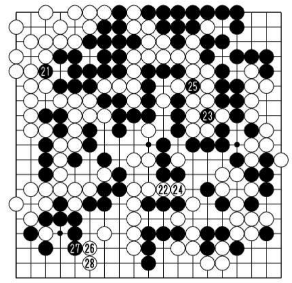


❶ as Black's 233
❸(take back at the left of ②)
❺(fill at the right of ❶)

Sequence of the Real Game



⑯(take at the right of ⓭)



White wins by 31.5 points.
Omitted after ㉘

Summary

- Semeai Game
  - Model of liberty counting in capturing races
  - New genre of application of CGT to the game of Go
- There are two key points:
  - Method of assigning liberty scores to terminal nodes
  - Cooling by 2 degrees to analyze
- Evaluation formula for semeai with non-plain external liberties
- Analysing kos in semeai