Evaluating territories of Go positions with capturing races

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Abstract

- On analyzing capturing races, we have been focusing on the method to find which player wins the race so far, because whether to win or to lose the race largely affects the territory score and it somtimes can decide the outcome of the game.
- But in order to evaluate the state of the game properly, we usually have to count the territory score precisely regardless of which player wins the race.
- Sometimes the loser of a capturing race has good moves, although the moves don't affect the status of winning or losing the race.
- We propose a method for evaluating territory score in each decomposed subgame of a capturing race considering the status of the winner of the race.

Contents

- Introduction
 - How to count liberties using CGT
 - ► Liberty Counting Game (LCG)
 - Assigning liberty scores to terminal nodes
 - ► Cooling by 2
 - Example problems of capturing races
- Evaluating territories with capturing races
 - Territory games on LCGs
 - semedori
- Summary

Introduction

Applications of CGT to the Game of Go

Endgame

Berlekamp, Wolfe: "Mathematical Go", (1994)

- Score is the sum of the number of points in territories and the number of prisoners. (Japanese rule)
- But we don't need to assign the final score explicitly.
- The score comes out of CGT itself by forbidding passes.
- Cooling by one degree, or *chilling*, makes us get the last point, *tedomari*.

Introduction

Applications of CGT to the Game of Go

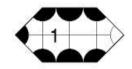
Eyespace Value

Landman: "Eyespace Values in Go", (1996)

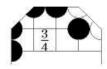
- Bargo: Eyespace Counting Game
 - ▶ The final score is the number of distinct eyes.
 - ▶ It doesn't matter how many points are contained in eye regions.
 - ▶ Chilling also works.



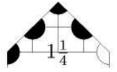


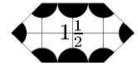












Introduction

A New Genre of Application of CGT to the Game of Go

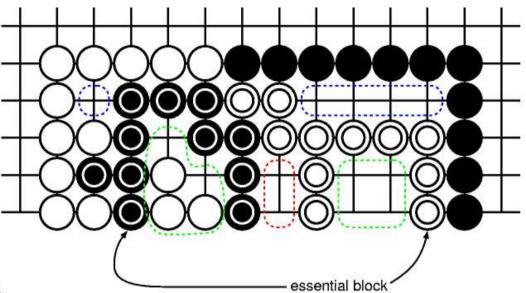
Counting Liberties in Capturing Races

Nakamura (2003):

"Counting Liberties in Capturing Races using Combinatorial Game Theory" (in Japanese), IPSJ SIG-GI Technical Report, 2003 GI-9-5

- CGT can be applied to any situations that involve counting.
- Capturing Race, or Semeai is a liberty counting game and is decomposable,
 - because for a semeai involving a big dragon we usually count liberties for each part of the dragon and sum up them.

Terminology for Semeai



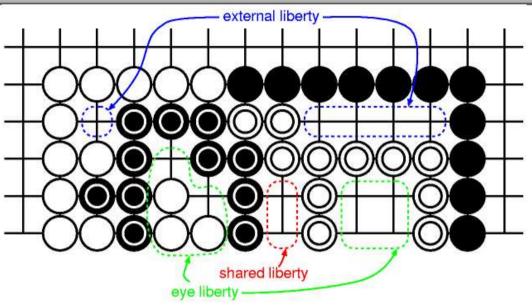
Essential block

- A block of black or white stones which must be saved from capture.
- Capturing an essential block immediately decides a semeai.

Liberty region

- A region which is surrounded by at least one essential block and some other essential blocks and safe blocks.
- A liberty region is called external region if its boundary does not consist of the essential blocks of different color.

Terminology for Semeai



■ External liberty

A liberty of an essential block which is in an external liberty region

Shared liberty

A common liberty of a Black's essential block and a White's essential block

Plain liberty

- Liberty of essential block which is also adjacent to the opponent's safe or essential block.
- Attacker can fill the plain liberty without making additional approach moves.

Terminology for Semeai

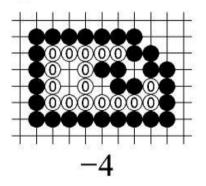
- Attacker / Defender
 - In an external liberty region, the player who owns the essential block is called *defender* and the opponent is called *attacker*
- Attack
 - to reduce liberties of the defender's essential block
 - ▶ Fill liberties one by one
 - ▶ Cut the defender's blocks to take away some more liberties
- Defend
 - to extend liberties of defender's own essential block
 - ► Connect to the other own blocks to gain the more liberties
 - Escape to a large empty region
 - Attack some weak points of attacker to force the more additional approach moves

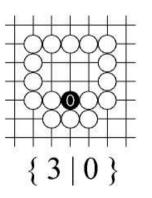
Counting Liberties using CGT

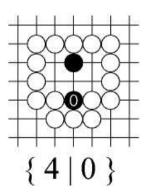
- Liberty Counting Game (LCG)
 - Score is the number of liberties of essential blocks
 - ▶ Liberty count is not always a number, but a conbinatorial game
 - Black's liberties are positive and White's are negative

$$G = \left\{ G^L \mid G^R \right\}$$

Examples:







■ It seems easy, but in fact, we have to resolve a subtle problem to get these descriptions.

How should we assign the terminal score?

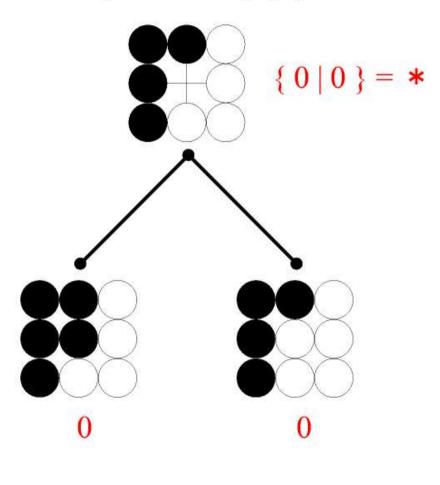
- In case of Go endgame,
 - We don't need to assign the terminal score explicitly.
 - The score comes out of CGT itself, if we forbid pass moves and permit to return one prisoner to the opponent as a move instead of playing on the board.
- In case of eyespace value,
 - We have to assign the terminal score as the number of distinct eyes.
 - Once defender makes a secure eye space, attacker cannot destroy it.

How should we assign the terminal score?

- But in case of counting liberties,
 - Attacker can always fill the defender's liberties one by one.
 - There are no secure liberties in semeai.
 - ► Even eye liberties are not secure.
 - Defender should not play a move into his own liberty region, if he cannot extend the liberty at least one.
 - It is always bad move to take away his own liberty!
- We have to prune the defender's bad move explicitly in contrast to endgame, in which pruning is performed implicitly by means of legality.

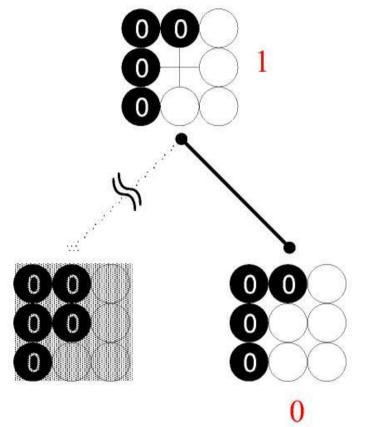
How should we assign the terminal score?

Stone putting game with just one empty point



How should we assign the terminal score?

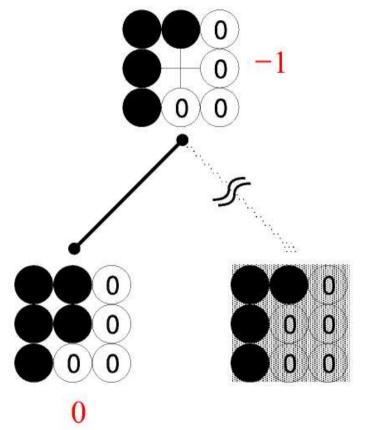
Liberty counting game (Black's essential block)



Pruning bad move which reduces his own liberty

How should we assign the terminal score?

Liberty counting game (White's essential block)



Pruning bad move which reduces his own liberty

Process for Assigning Terminal Liberty Score

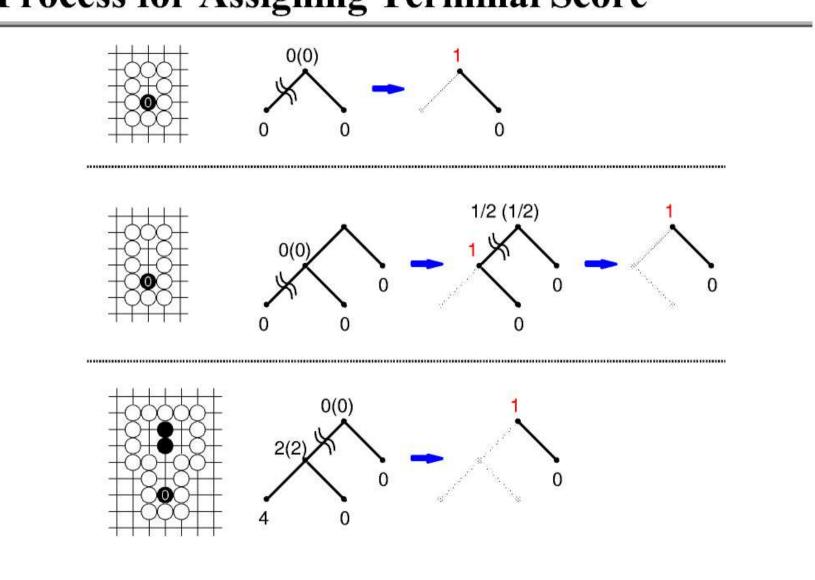
- Play out all legal moves for both players until the essential block has no liberty
 - ▶ At the present, it doesn't matter whether the move is good or bad.
 - All the leaf nodes are zero.
 - ▶ In practice, however, we can count the number of liberties when they become plain liberties.
- From bottom to top, execute the following operations
 - ▶ If the temperature of a node is less than or equal to 1, prune the defender's branch.
 - ▶ If the resulting node becomes either of the following, replace it.

$$\{ |n \} (n \ge 0) = > n+1$$

 $\{-n | \} (n \ge 0) = > -n-1$

(This replacement operation is exactly contrary to conventional CGT)

Process for Assigning Terminal Score



BIRS CGT Workshop 2008/01/22 **Process for Assigning Terminal Score** -BIG

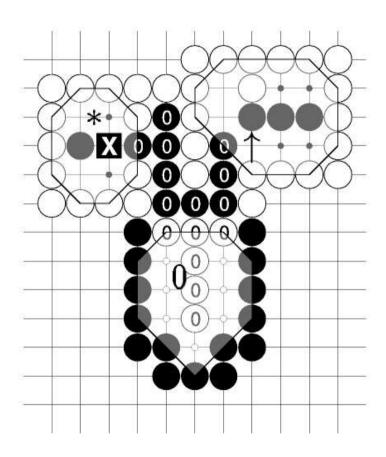
Semeai Games

- Sum of liberty counting games (Sum of LCGs)
- Assumed that each summand has just one essential block.
- Each summand is preprocessed and assigned terminal scores using the procedure mentioned before.
- Rules for play are same as Go. Suicidal moves are forbidden except for the last winning move.
- Player who fills all of the liberties of opponent's all essential block in all summands is a winner.
- In case of endgame and eyespace value, the smallest incentive of move is *0-ish*, almost zero, and chilling works well.
- In case of semeai, the smallest incentive is *1-ish*, because the attacker can always fill the opponent's liberties one by one.
- So, cooling by 2 degrees must work to analyze semeai games.

Evaluation Method of Semeai Game

Supposed that G is a semeai game and g is Cool(G, 2).

- Case 1: g is an integer:
 - If g > 0, Black wins.
 - If g < 0, White wins.
 - If g = 0, first player wins.
- **Case 2:** n+1 > g > n (for some integer n):
 - If Black plays first, he can round g up to n+1 in keeping his turn.
 - If White plays first, he can round g down to n in keeping his turn.
 - Check the resulting adjustment value using the conditions of case 1.
- Case 3: g <> n (for some integer n):
 - If Black plays first, he can round g up to n+1 in keeping his turn.
 - If White plays first, he can round g down to n−1 in keeping his turn.
 - Check the resulting adjustment value using the conditions of case 1.



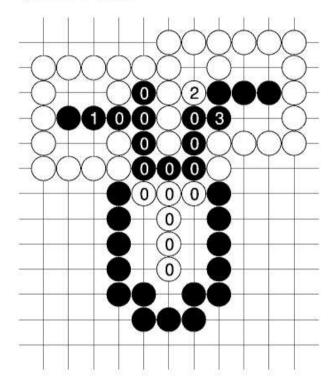
$$\begin{cases}
4 \mid 0 \end{cases} \stackrel{\text{cool by 2}}{\Longrightarrow} 2 * \\
\begin{cases}
6 \mid \{4 \mid 0\} \} \stackrel{\text{cool by 2}}{\Longrightarrow} 4 \uparrow \\
-7
\end{cases}$$

$$Total = -1 \uparrow * <> -1$$

- Black's adjustment value is 0.
- Black can win, if he plays first.

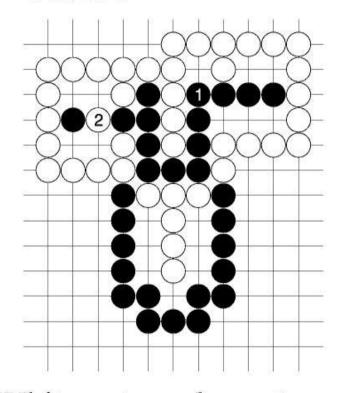
Problem 1

Success

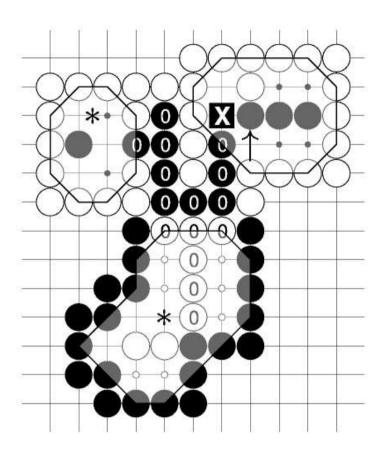


Black gets tedomari

■ Failure



White gets tedomari



$$\{4 \mid 0\} \stackrel{\text{\tiny cool by 2}}{\Longrightarrow} 2 *$$

$$\{6 \mid \{4 \mid 0\}\} \stackrel{\text{\tiny cool by 2}}{\Longrightarrow} 4 \uparrow$$

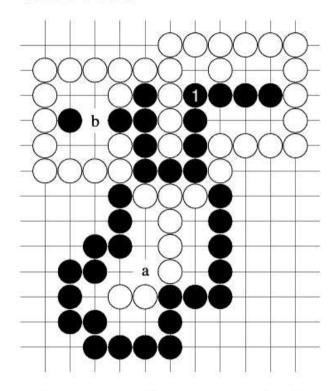
$$\{-5 \mid -9\} \stackrel{\text{\tiny cool by 2}}{\Longrightarrow} -7 *$$

$$Total = -1 \uparrow > -1$$

- Black's adjustment value is 0.
- Black can win, if he plays first.

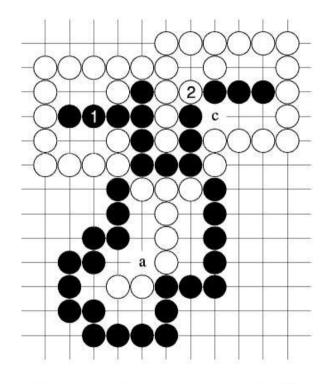
Problem 2

Success

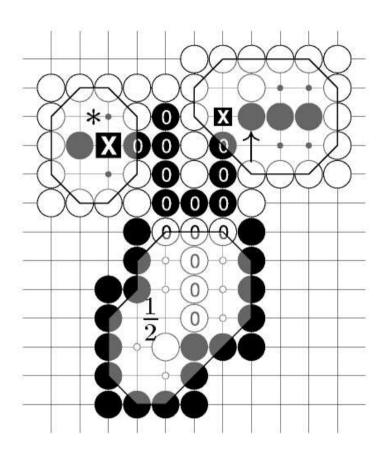


(a and b are miai) Black gets tedomari

■ Failure



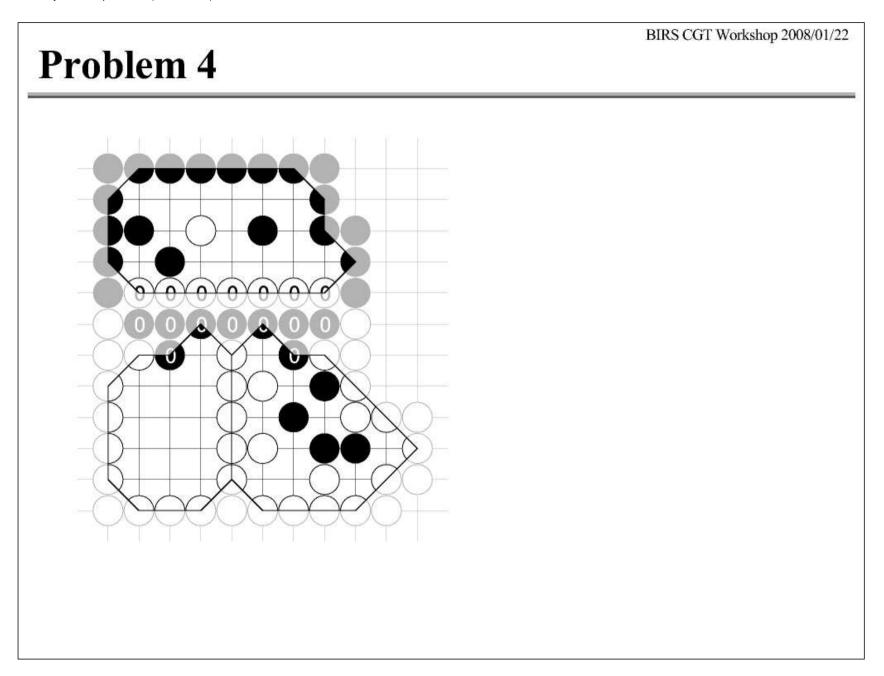
(a and c are miai) White gets tedomari

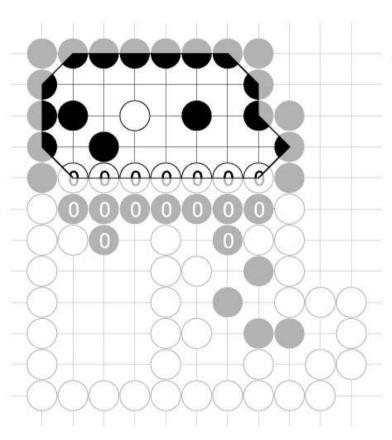


$$\begin{cases}
4 \mid 0 \end{cases} \stackrel{\text{cool by 2}}{\Longrightarrow} 2 * \\
\begin{cases}
6 \mid \{4 \mid 0\} \} \stackrel{\text{cool by 2}}{\Longrightarrow} 4 \uparrow \\
\begin{cases}
-5 \mid -8 \end{cases} \stackrel{\text{cool by 2}}{\Longrightarrow} -7 + \frac{1}{2} \\
\end{aligned}$$

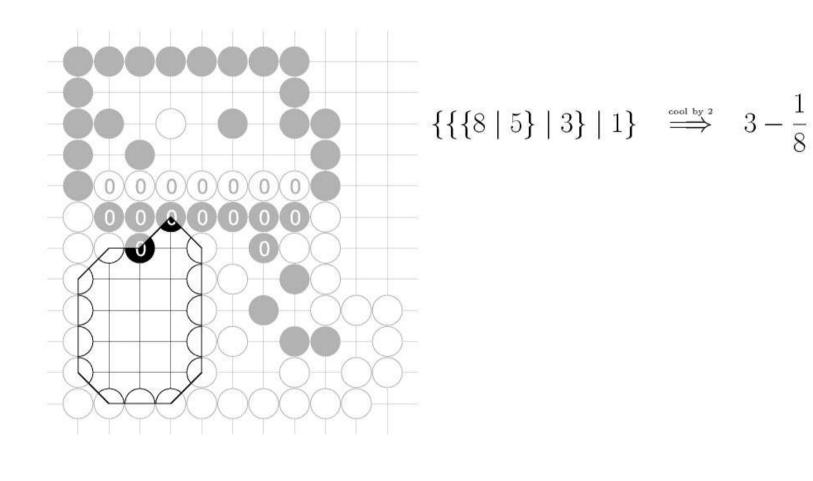
$$\text{Total} = -\frac{1}{2} \uparrow * > -1$$

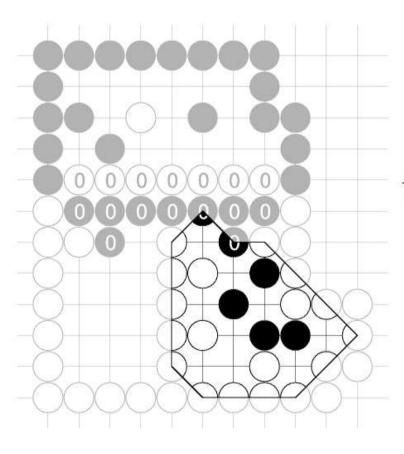
- Black's adjustment value is 0.
- Black can win, if he plays first.



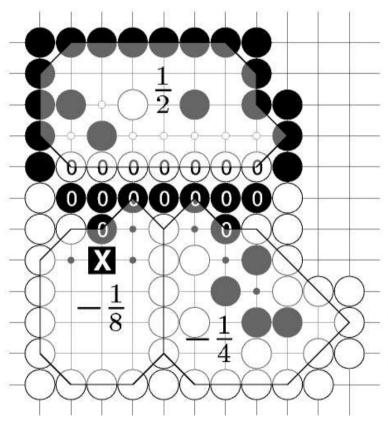


$$\{-5 \mid -8\} \quad \stackrel{\text{\tiny cool by 2}}{\Longrightarrow} \quad -7 + \frac{1}{2}$$





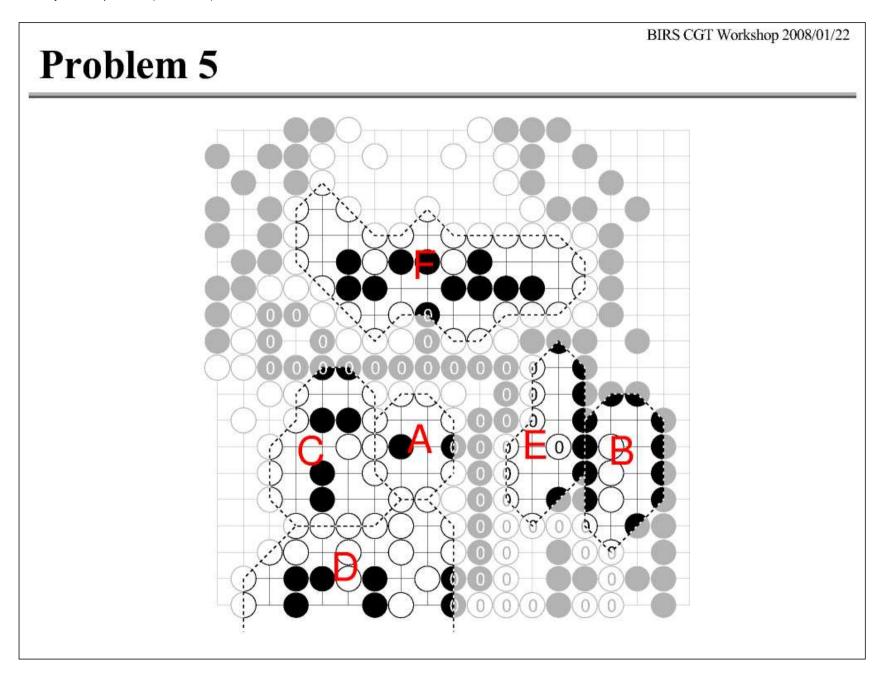
$$\{\{6 \mid 3\} \mid 1\} \quad \stackrel{\text{\tiny cool by 2}}{\Longrightarrow} \quad 3 - \frac{1}{4}$$



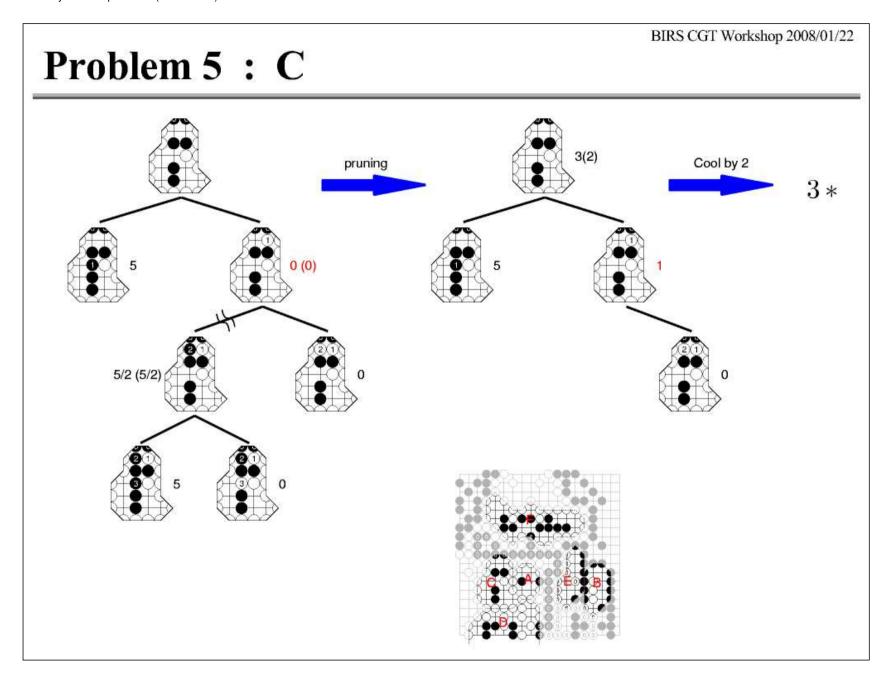
$$Total = -\frac{7}{8} > -1$$

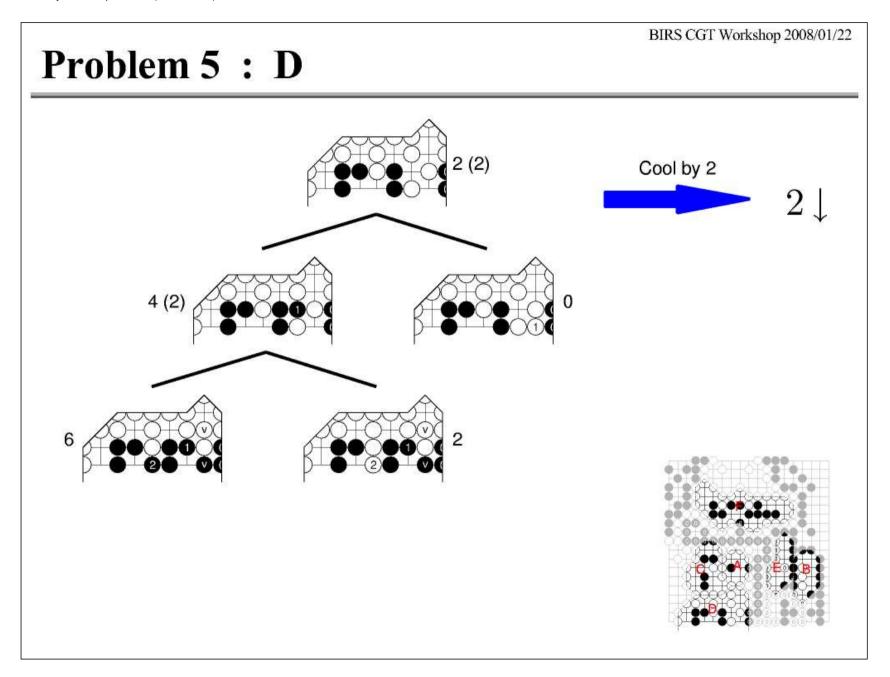
- Black's adjustment value is 0.
- Black can win, if he plays first.

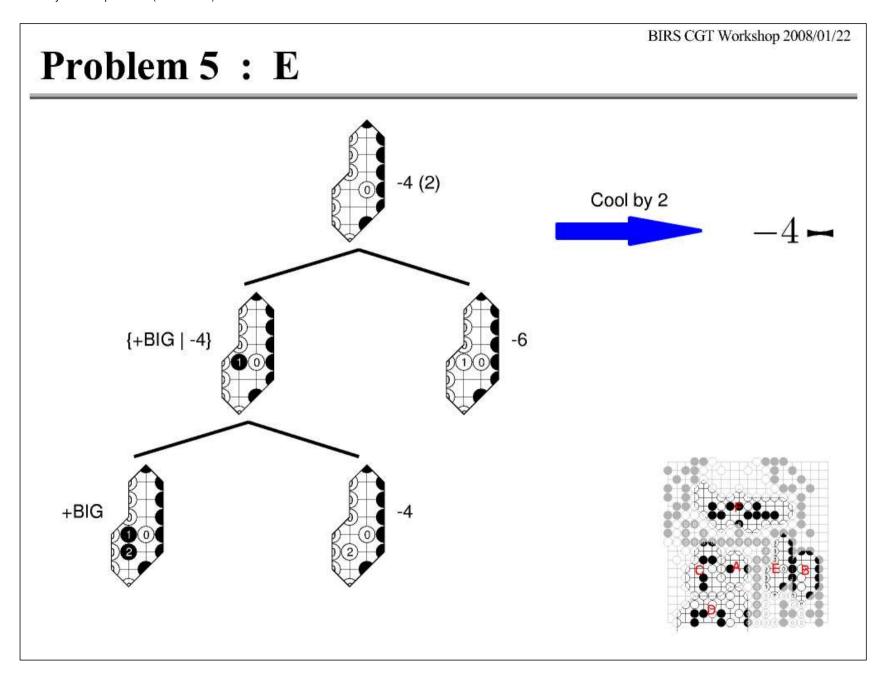
BIRS CGT Workshop 2008/01/22 Problem 5 (whole board problem) 0000000000



BIRS CGT Workshop 2008/01/22 Problem 5: A, B Cool by 2 2(2) 2 * Cool by 2 -2(2)

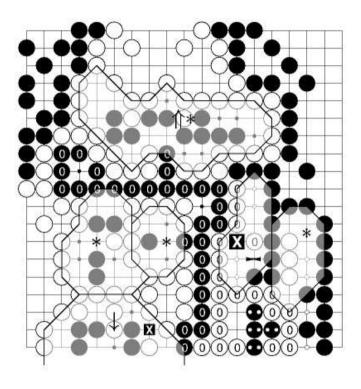






BIRS CGT Workshop 2008/01/22 Problem 5: F 6(2)Cool by 2 $6 \uparrow *$ 4(2)

Problem 5



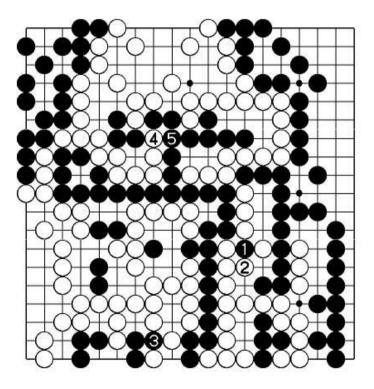
Total =
$$(2*) + (-2*) + (3*) + (2\downarrow) + (-4 -) + (6 \uparrow *) + 2 - 10 = -1 \uparrow -$$

Range: $0 > -1 \uparrow - > -1$

• Black's adjustment value is 0 ===> Black can win

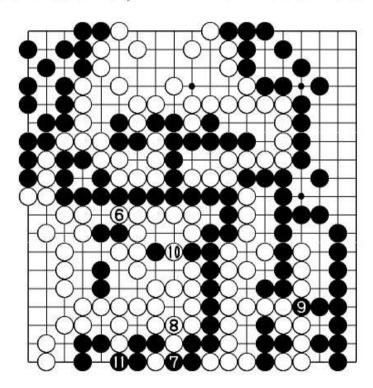
Correct sequence

- B1 (throw in) at first, then B3.
- B5 in response to W4 is important.



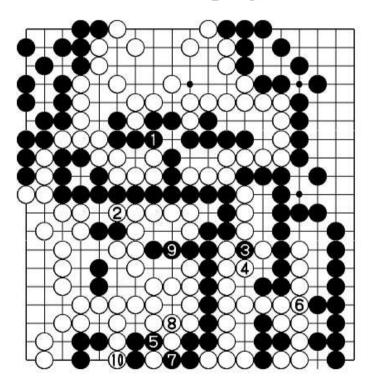
Correct sequence

- All of W6, B9, W10 and B11 are same.
- Actual battle ends here and Black wins the semeai.
- Black's liberties: 15, White's liberties: 14



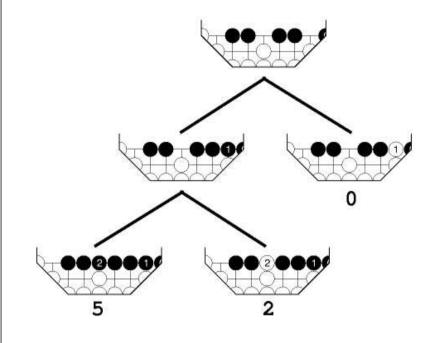
Failure

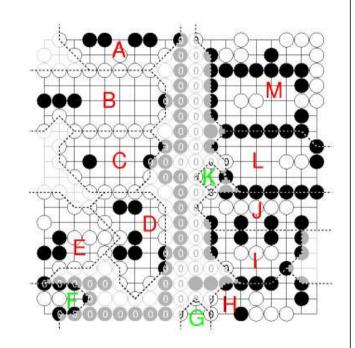
- If Black plays B1, White plays W2 and Black cannot win the semeai even if Black plays B3 and B5.
- B1 is a fatal error that 'Left plays to UP'.



BIRS CGT Workshop 2008/01/22 Problem 6 (whole board problem)

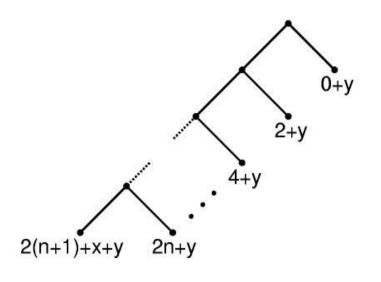
Problem 6 : A

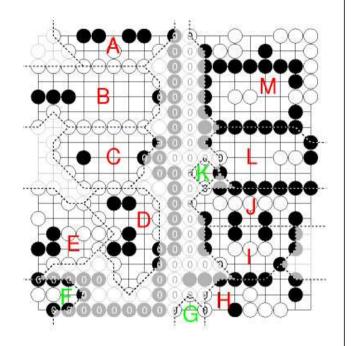




$$\{\{5 \mid 2\} \mid 0\} \quad \stackrel{\text{\tiny cool by 2}}{\Longrightarrow} \quad 1 \frac{3}{4}$$

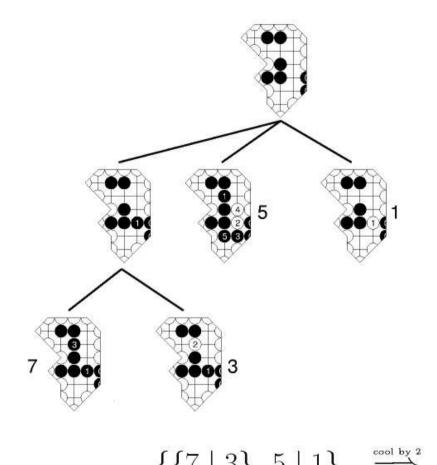
Problem 6: B, C, -L

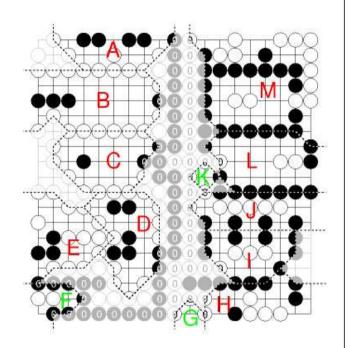




- B n = 4, x = 5, y = 0 {{{{{15 | 8} | 6} | 4} | 2} | 0} \Longrightarrow 2 {-3 | 0³}
- C n=2, x=3, y=1 {{{10 | 5} | 3} | 1} $\stackrel{\text{cool by }^2}{\Longrightarrow}$ 3 {-1 | 0}
- -L n=2, x=4, y=1 {{{11 | 5} | 3} | 1} $\stackrel{\text{cool by 2}}{\Longrightarrow}$ 3 {-2 | 0}

Problem 6: D



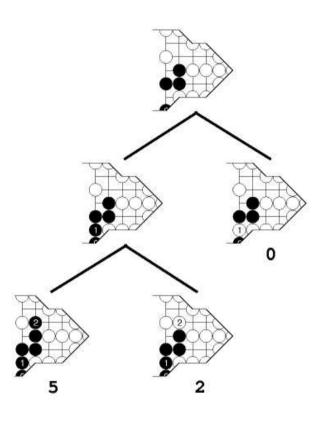


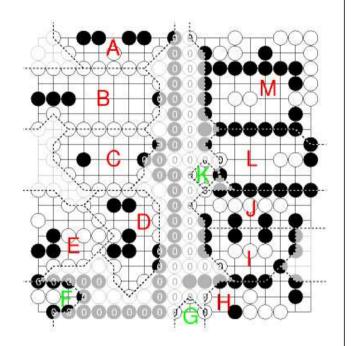
$$\{\{7 \mid 3\}, 5 \mid 1\}$$

$$\stackrel{\text{cool by 2}}{\Longrightarrow}$$

$$\{\{7 \mid 3\}, 5 \mid 1\} \stackrel{\text{\tiny cool by 2}}{\Longrightarrow} 3 \{0, * \mid 0\} = 3 \uparrow *$$

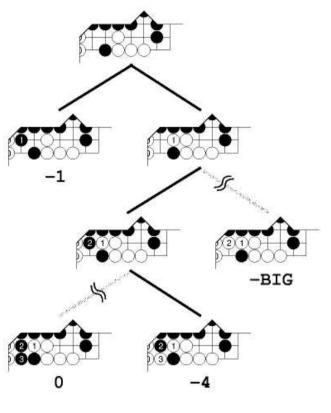
Problem 6 : E

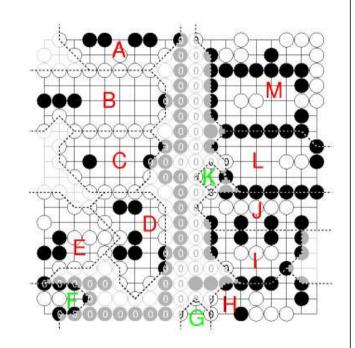




$$\{\{5 \mid 2\} \mid 0\} \quad \stackrel{\text{\tiny cool by 2}}{\Longrightarrow} \quad 1$$

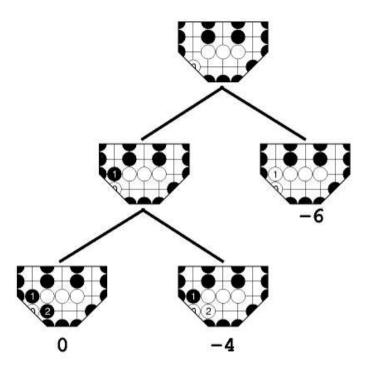
Problem 6: H

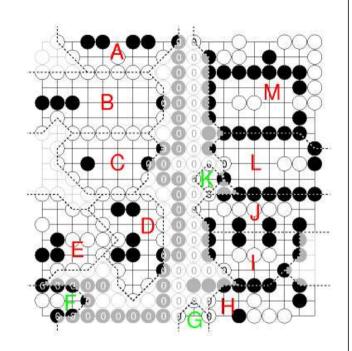




$$\{-1 \mid -4\} \stackrel{\text{\tiny cool by 2}}{\Longrightarrow} -2\frac{1}{2}$$

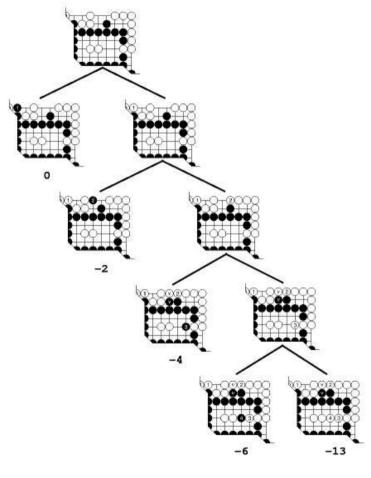
Problem 6: I

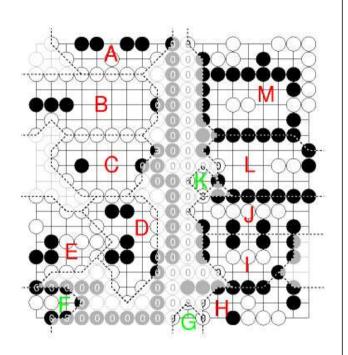




$$\{\{0 \mid -4\} \mid -6\} \quad \stackrel{\text{\tiny cool by 2}}{\Longrightarrow} \quad -4 \downarrow$$

Problem 6: M



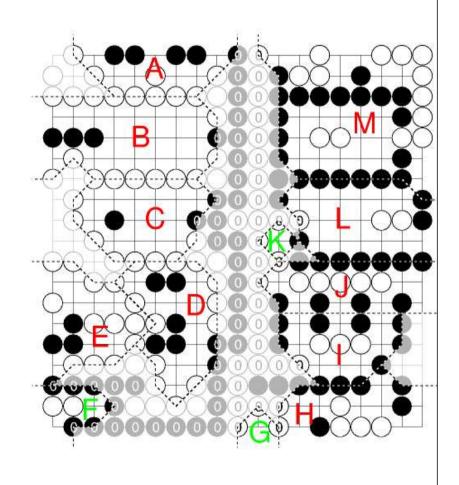


$$\{0 \mid \{-2 \mid \{-4 \mid \{-6 \mid -13\}\}\}\}\ \stackrel{\text{\tiny enerl by 2}}{\Longrightarrow} \ -2 \ \{0^2 \mid {}_{+3}\}$$

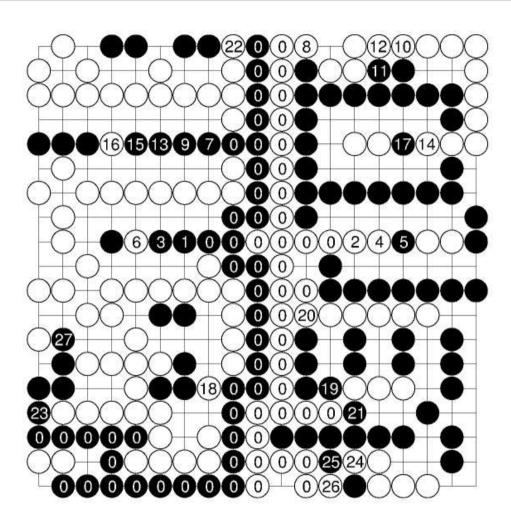
Problem 6

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	value	atomic weight
A	$1\frac{3}{4}$	0
В	$2\left\{{3}\mid 0^{3}\right\}$	-3
C	$3\{ \leftarrow_1 \mid 0 \}$	-1
D	3↑*	1
E	$1\frac{3}{4}$	0
F	3	0
G	-1	0
H	$-2\frac{1}{2}$	0
I	$-4 \downarrow$	-1
J	-2*	0
K	-1	0
L	$-3\{0 \mid \mathbf{+}_2\}$	1
М	$-2\{0^2 \mid \mathbf{+}_3\}$	2
Total	-1 ish	:-1,,,



Problem 6: Answer



- Example continuation up to B 27
 - B: 22 liberties
 - W: 21 liberties
- Black wins by one move ahead

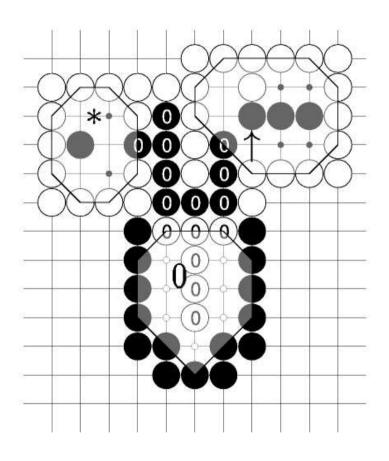
Summary for analyzing semeai

- Semeai Game
 - Model of liberty counting in capturing races
 - New genre of application of CGT to the game of Go
- Two key points in applying CGT to capturing races:
 - Method of assigning liberty scores to terminal nodes
 - Cooling by 2 degrees works to analyze

Toward the next step

- Whether to win or to lose a capturing race largely affects the territory score and it somtimes can decide the outcome of the entire game.
- But in order to evaluate the state of the game properly, we usually have to count the territory score precisely regardless of which player wins the race.
- Sometimes the loser of a capturing race still has good moves, although the moves don't affect the status of winning or losing the race.
- We propose a method for evaluating territory score in each decomposed subgame of a capturing race considering the outcome of the race.

Semeai Problem 1 (revisited)



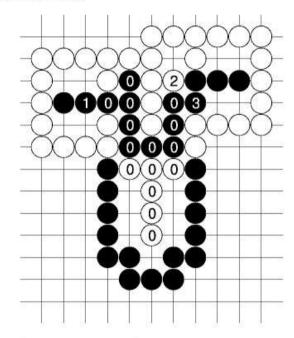
$$\begin{cases}
4 \mid 0 \end{cases} \stackrel{\text{cool by 2}}{\Longrightarrow} 2 * \\
\begin{cases}
6 \mid \{4 \mid 0\} \} \stackrel{\text{cool by 2}}{\Longrightarrow} 4 \uparrow \\
-7
\end{cases}$$

$$Total = -1 \uparrow * <> -1$$

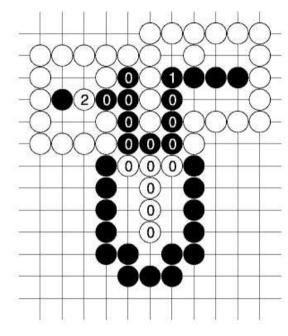
- Black's adjustment value is 0.
- White's adjustment value is -2.
- First player wins.

Black's winning sequence

Success



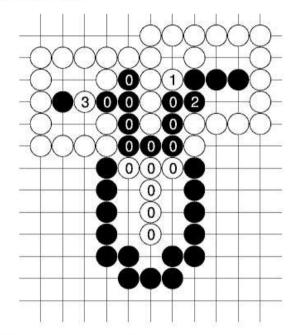
Black gets *tedomari* B = 8, W = 7Black wins by 1 move Failure



White gets *tedomari* B = 6, W = 7 White wins by 1 move

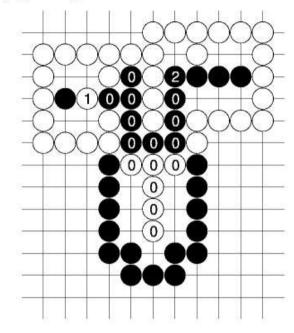
White's winning sequence

Success



White gets *tedomari* B = 4, W = 7 White wins by 3 moves

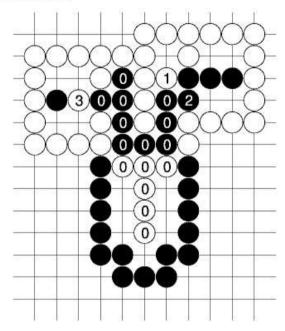
(Quasi) Success



Black gets *tedomari* B = 6, W = 7White wins by 2 moves (with a White's extra move)

Territory Score (White plays first)

Success

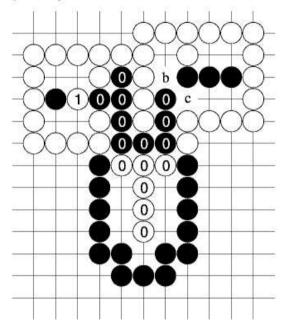


- Total: 33 points (for White)
 - Upper Left: −6
 - Upper Right: −1
 - Lower: 0
 - Body: -26

(Black's 13 stones)

Territory Score (White plays first)

(Quasi) Success



- Although Black loses the race, Black b is an effective move.
 - ==> semedori

- Black b
 - Total: 32 points (for White)
 - ► Upper Left: -6
 - ▶ Upper Right: (
 - ▶ Body: -26
- White b, Black c
 - Total: 33 points (for White)
 - ► Upper Left: -6
 - ► Upper Right: -1
 - ▶ Body: -26
- White b, White c
 - Total: 35 points (for White)
 - ▶ Upper Left: −6
 - ► Upper Right: -11
 - ▶ Body: -18

Evaluating Territories

- Evaluating territories taking into account the winner of capturing races
 - Evaluate a territoriy score for each decomposed position
 - Sum of each score
- Three kinds of games: Glc, Gwin, Glose
 - ▶ Glc : Liberty counting game
 - ► Gwin: A game whose score is partial territory score, supposed that the defender (player who owns the essential block in the region) is the winner of an entire capturing race
 - Glose: A game whose score is partial territory score, supposed that the defender (player who owns the essential block in the region) is the loser of an entire capturing race

Evaluating Territories

- Glc is used to judge the outcome of capturing races
- Gwin or Glose are used to evaluate the territory score depending on who wins the race.
- Conditional Combinatorial Game (CCG)
 - Martin Mueller: "Conditional combinatorial games and their application to analyzing capturing races in Go", Information Sciences, 154, (2003)
 - Combine subgames with dependency

Conditional Combinatorial Game (CCG)

$$G \stackrel{\text{\tiny def}}{=} \{L_{C_1}^1, L_{C_2}^2, L_{C_3}^3, \dots \mid R_{D_1}^1, R_{D_2}^2, R_{D_3}^3, \dots \}$$

 L^i : left options of G

 R^i : right options of G

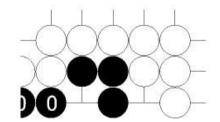
 C_i, D_i : context condition

(the truth value is determined from global game status)

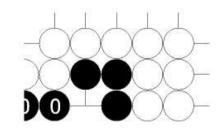
Example

$$G_3 = \{L_2 \mid 0_{L_0}, G_2\}$$
 $G_2 = \{L_1 \mid 0_{L_0}, G_1\}$ $G_1 = \{\mid 0\}$

$$G_2 = \{ L_1 \mid 0_{L_0}, G_1 \}$$



$$G_1 = \{ \mid 0 \}$$



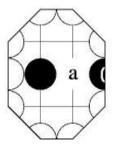
Conditional Combinatorial Game (CCG)

- How to use CCG for evaluating territory for each subgame?
 - Context condition : outcome of semeai game (sum of Glc)
 - Gwin and Glose are described as CCG
 - ===> pruning according to the result of Glc

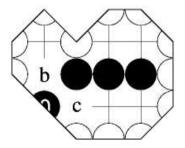
Example of Glc, Gwin, Glose

■ Subgame: A, B, C, D

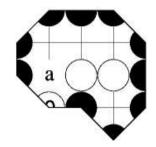
A: 2*

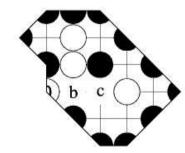


B: 4 1

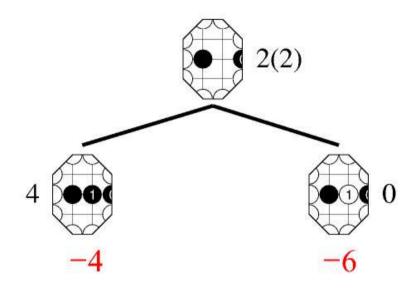


C : -2 *



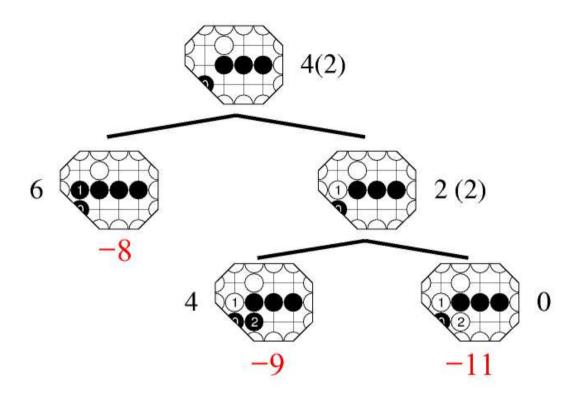


Subgame: A



• Glose: Defender (Black) is the loser

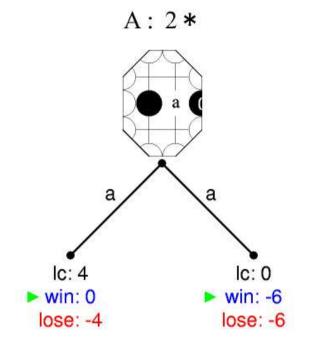
Subgame: B

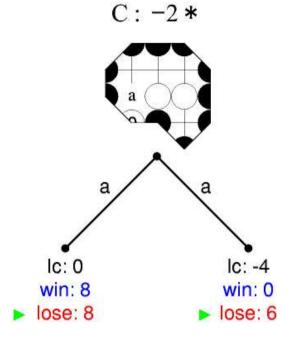


• Glose: Defender (Black) is the loser

BIRS CGT Workshop 2008/01/22 Subgame: A, C A: 2* C: -2 *lc: 4 lc: 0 Ic: 0 lc: -4 win: 0 win: 0 win: -6 win: 8 lose: -4 lose: -6 lose: 8 lose: 6

Subgame: A, C

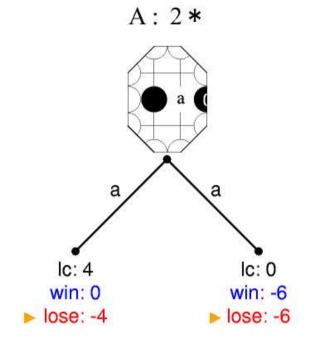


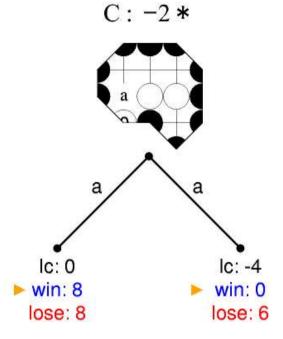


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in case that Black is the winner

Subgame: A, C

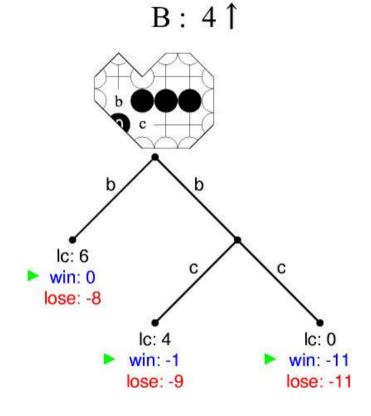


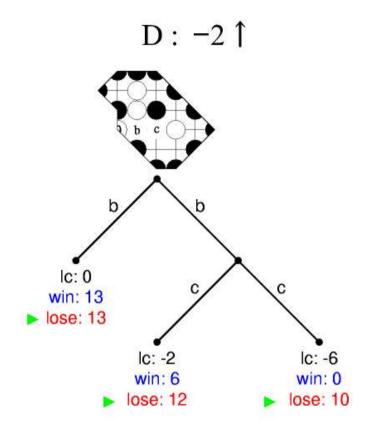


in case that White is the winner

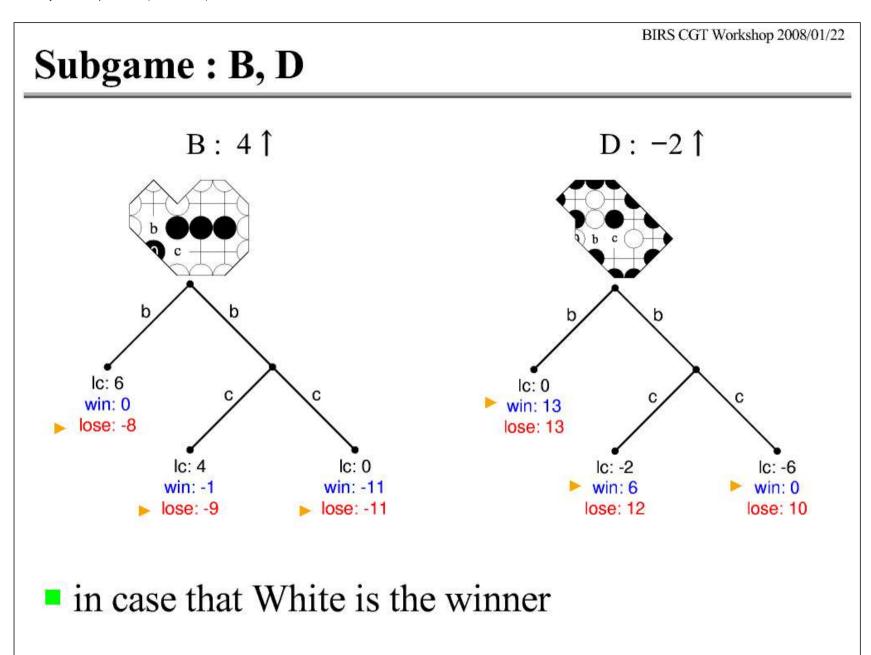
Subgame: B, D

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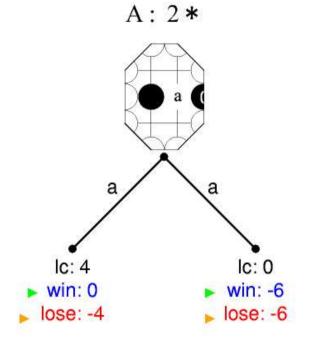


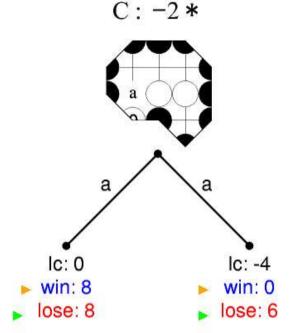


in case that Black is the winner



Compare A with C

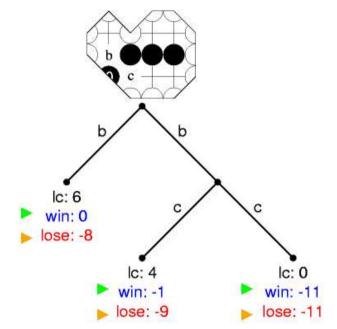




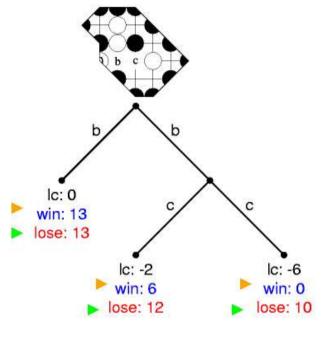
- if Black is the winner (▶): A is hotter than C
- if White is the winner (>): C is hotter than A

Compare B with D





D: -2 1



- if Black is the winner (▶): B is preferred to D
 - $(B \Rightarrow Tiny, D \Rightarrow \uparrow)$
- if White is the winner (▶) : D is hotter than B

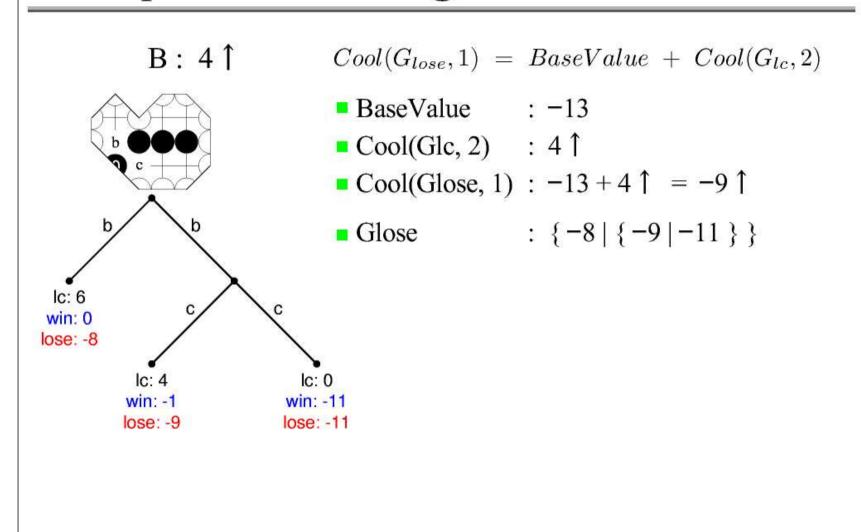
How to calculate the value of Glose

- Glose
 - Entire region should be a territory of the attacker
 - All of the defender's moves in this region are eventually captured by the attacker
- Formula to calculate the cooled value of Glose

$$Cool(G_{lose}, 1) = BaseValue + Cool(G_{lc}, 2)$$

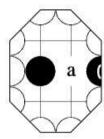
- BaseValue: Territory value supposed that the region is not involved in semeai and all the opponent blocks are dead.
- Cool(Glc, 2): Liberty count of the essential block number of attacker's moves in the region in order to capture the essential block
 - Attacker must fill his own territory by this number of moves eventually

Example for calculating the value of Glose

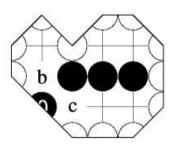


Semeail (1)

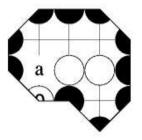
A: 2*



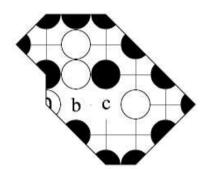
B: 4↑



C: -2 *



D: -2 1



Combine these subgames

$$A + B - 7 = (2 *) + (4 \uparrow) + (-7) = -1 \uparrow *$$

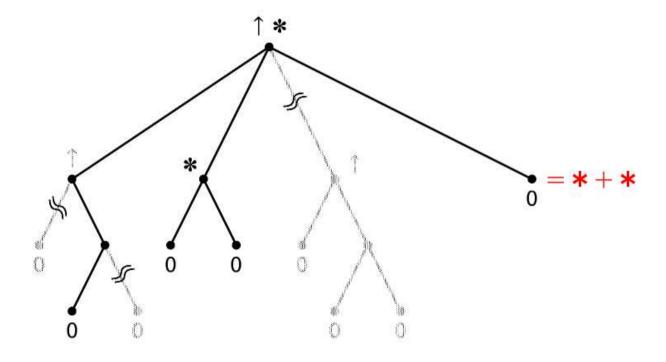
• 2:
$$A + D - 1 = (2 *) + (-2 \uparrow) + (-1) = -1 \uparrow *$$

• 3:
$$C + B - 3 = (-2 *) + (4 \uparrow) + (-3) = -1 \uparrow *$$

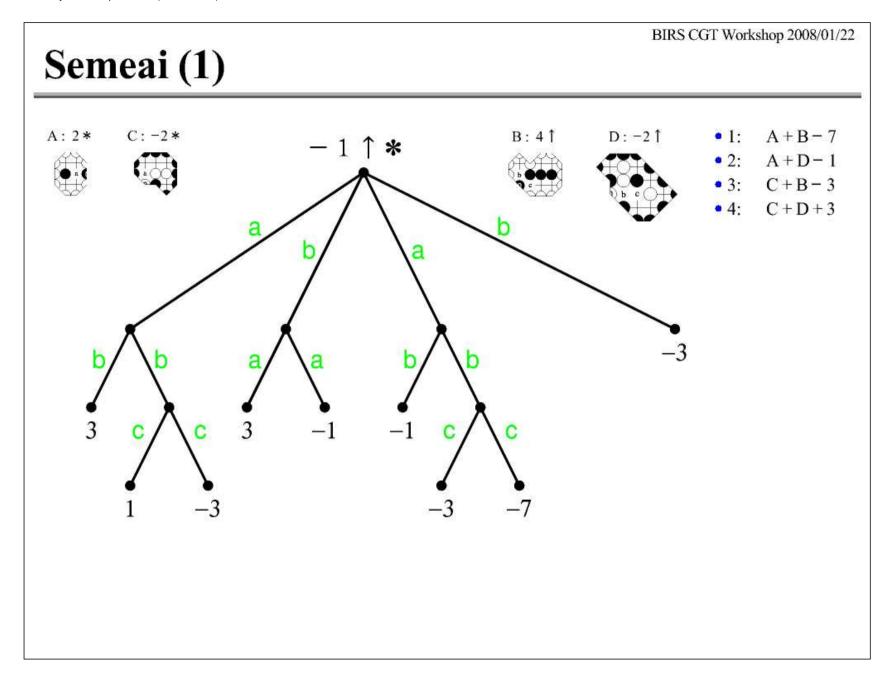
• 4:
$$C + D + 3$$

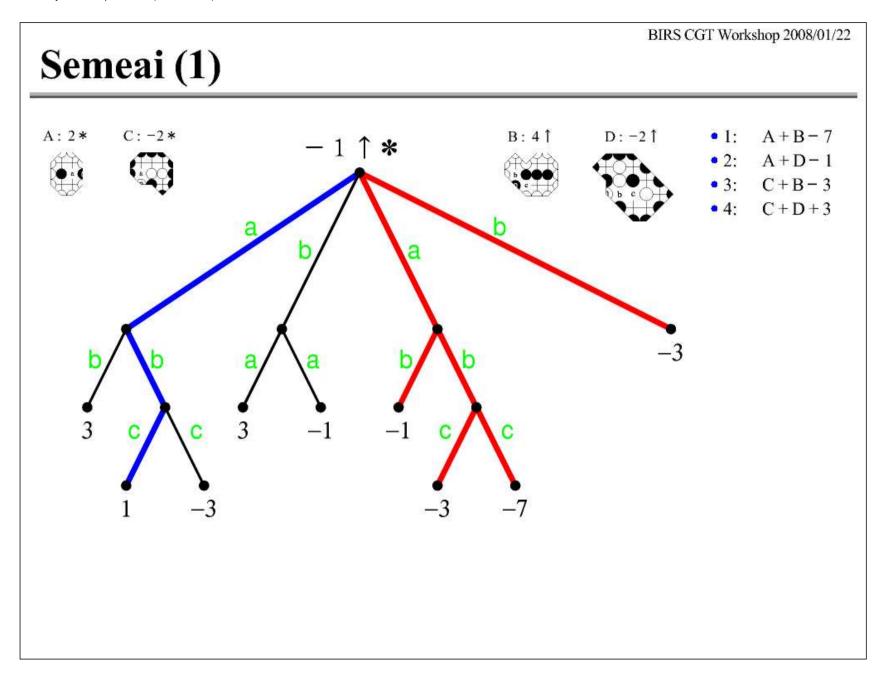
$$=> (-2*) + (-2 \uparrow) + (3) = -1 \uparrow *$$

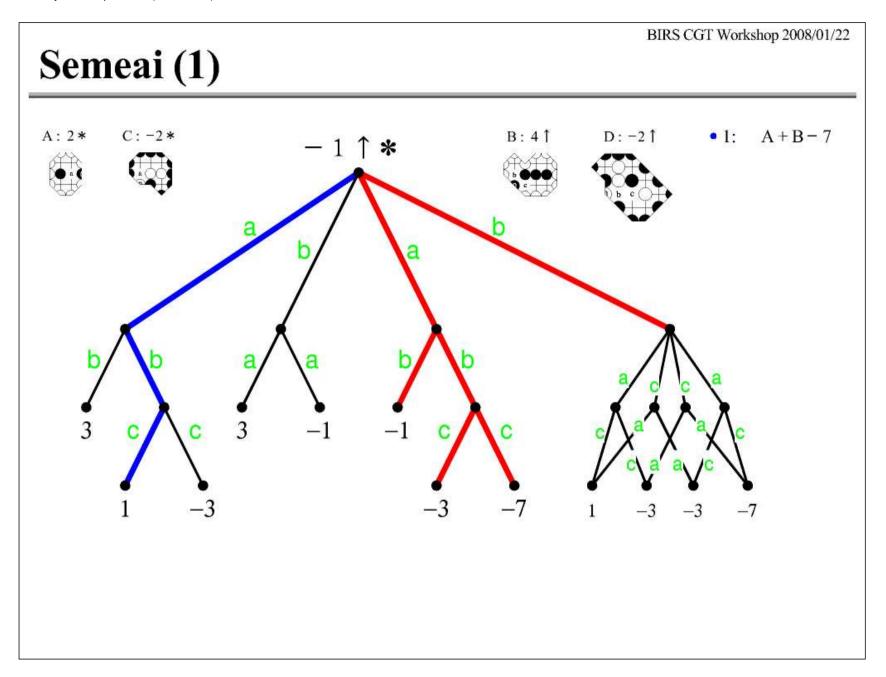
Canonical Game Tree of 1 *

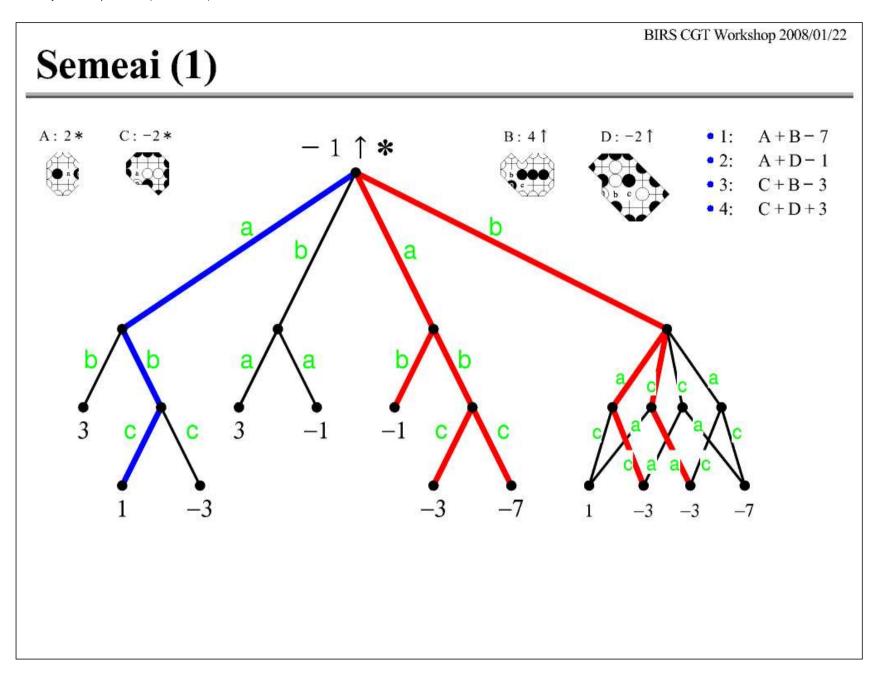


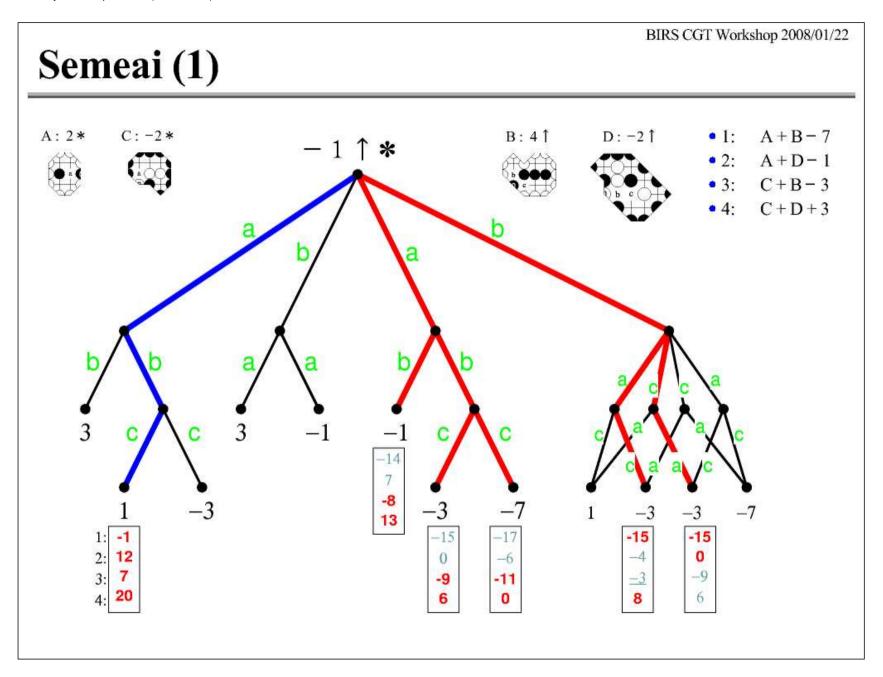
- $^{\bullet}$ \uparrow * = {0, * | 0}
- But, it's not true in semeai games (sum of Gls)
- It's enough for the attacker to win the race, because whether to win or to lose usually makes big difference in territory scores.

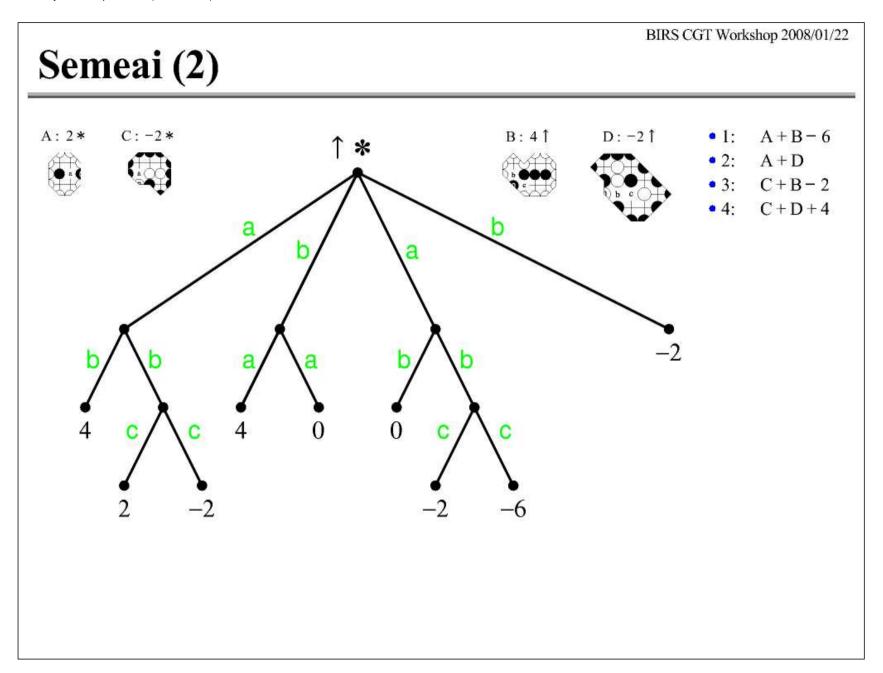


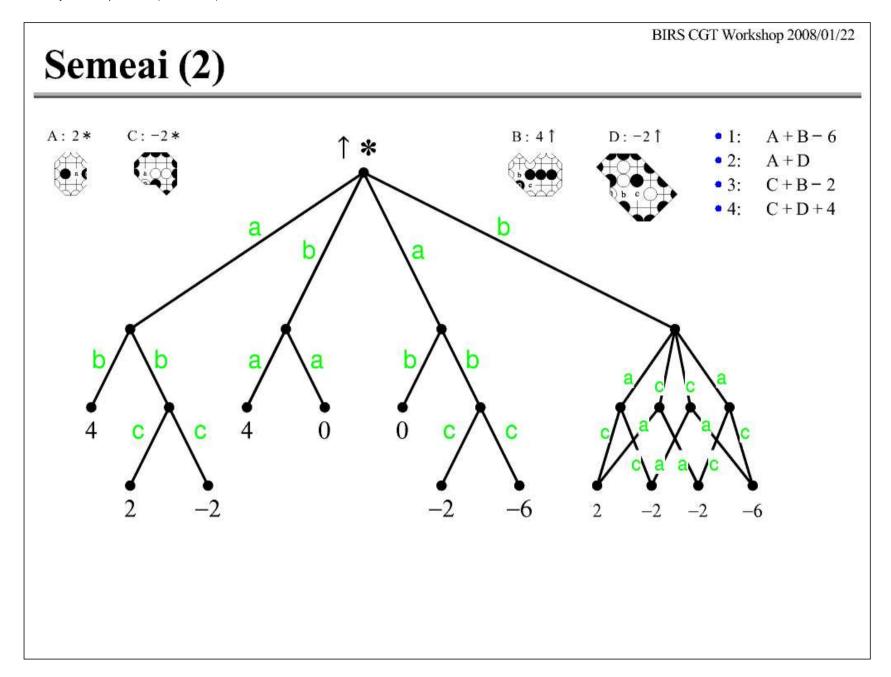


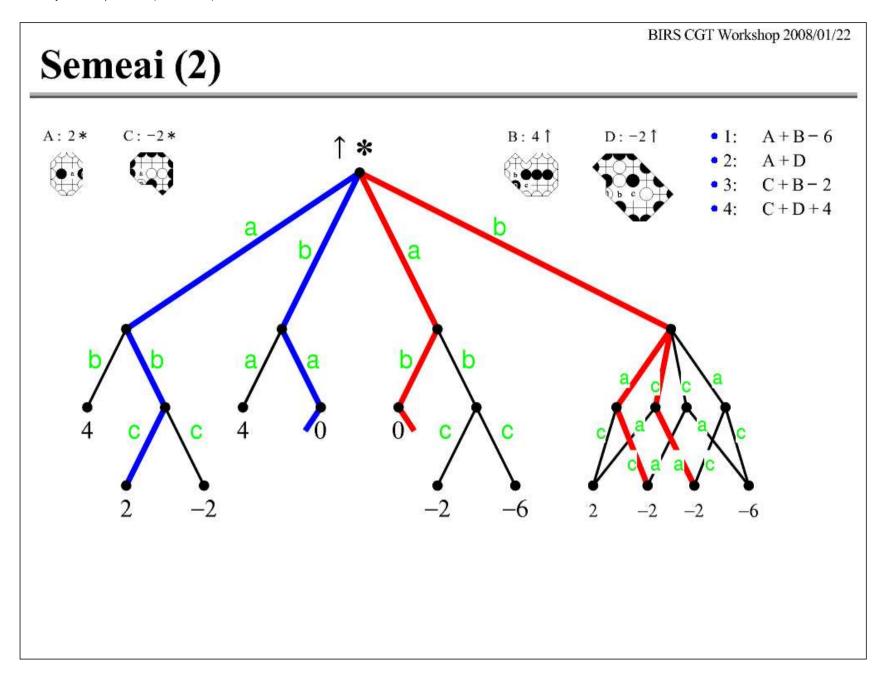


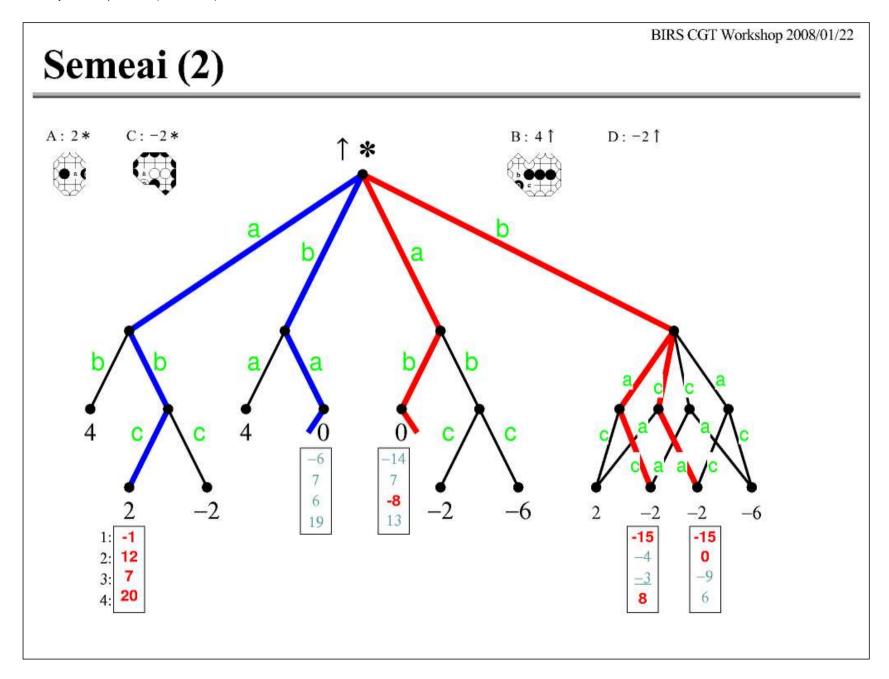


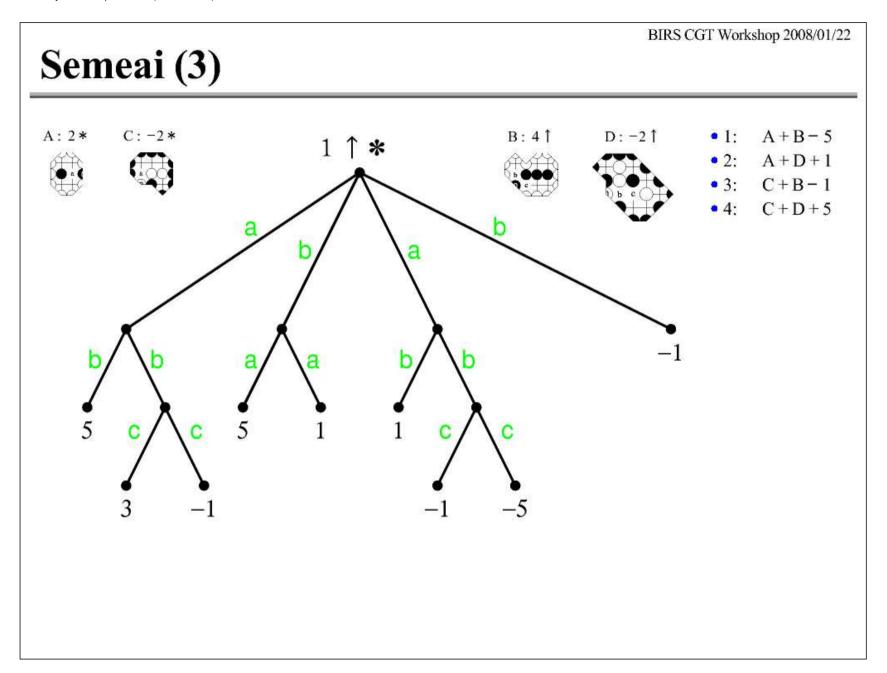


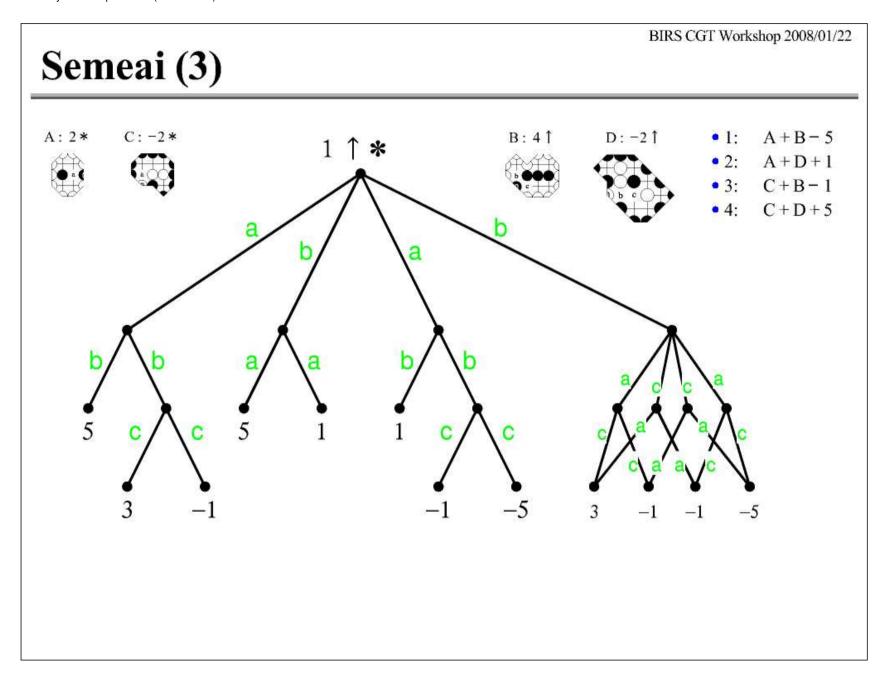


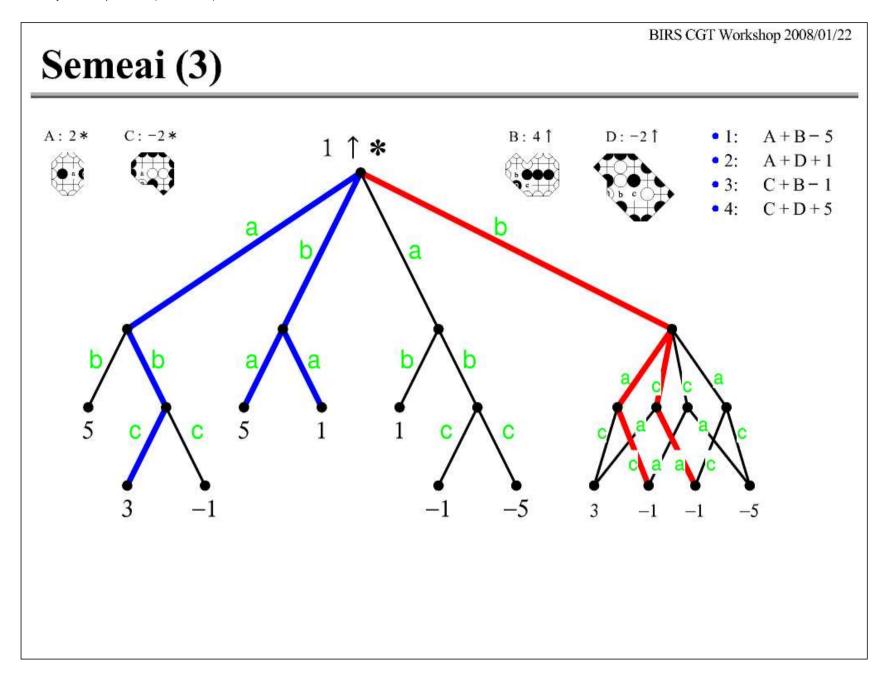


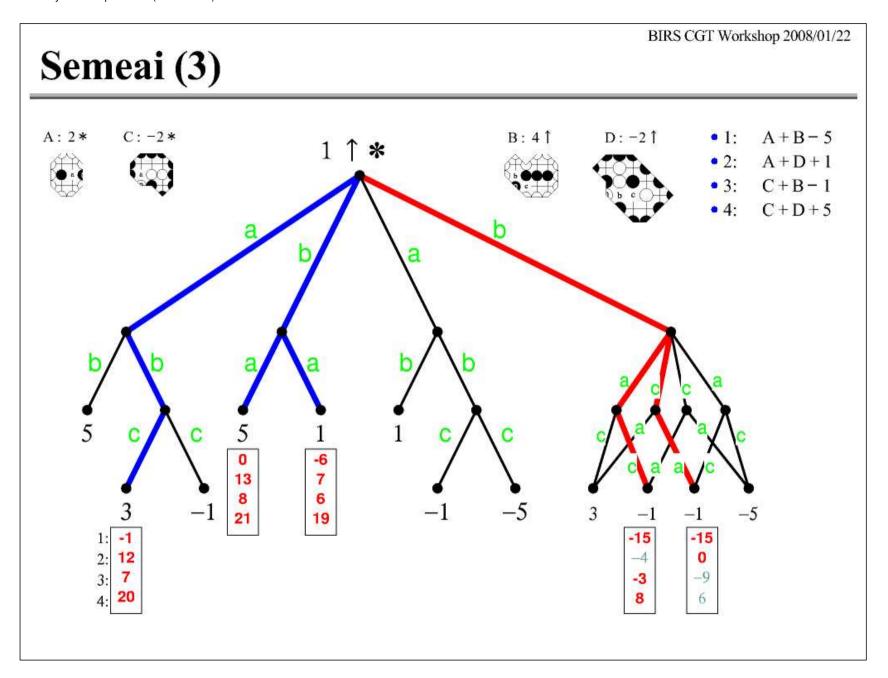


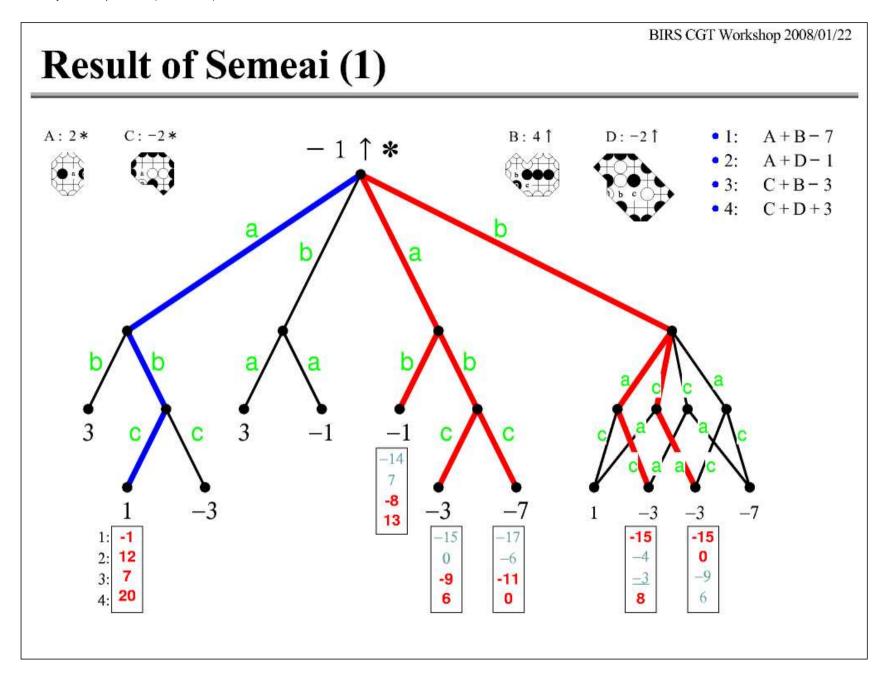


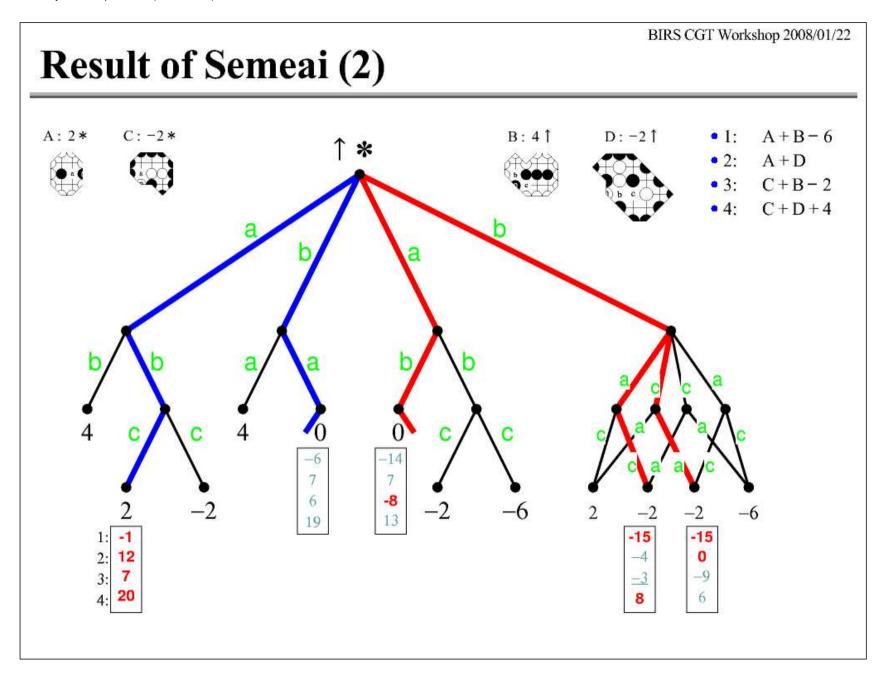


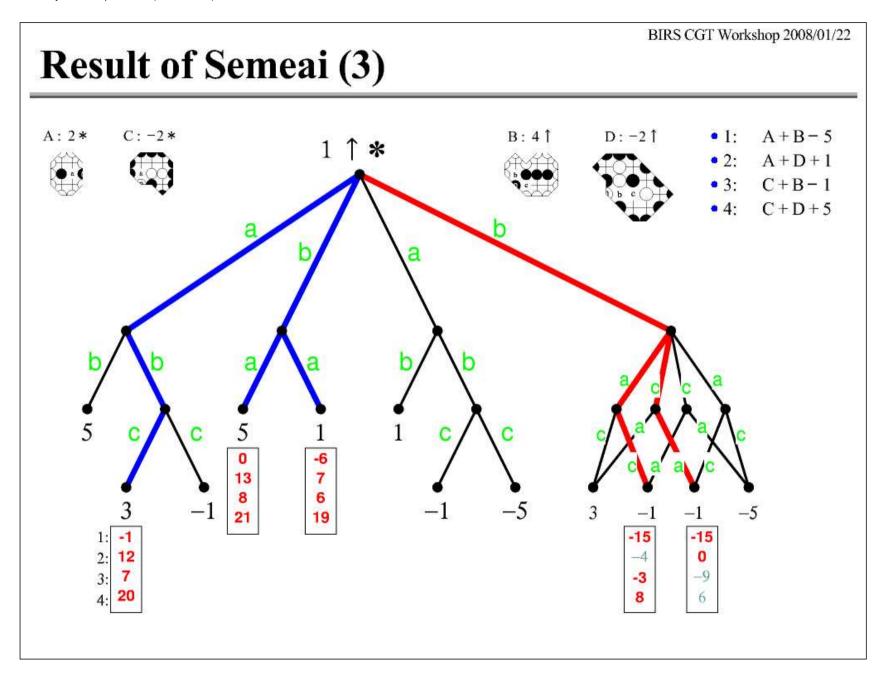












Summary

- Method for evaluating territory in consideration of the winner and the loser of capturing races
 - Three kinds of games should be considerd simultaneously
 - ▶ Glc, Gwin, Glose
 - Formula to calculate the cooled value of Glose
 - ► Cool(Glose, 1) = BaseValue + Cool(Glc, 2)
 - Evaluate Gwin or Glose depending on who win the entire semeai, that is the sum of Glc
 - But it's not easy!