A Method for Analyzing Complex Go Capturing Races

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Introduction

Application of CGT to Go capturing races

- Liberty counting game
- Assigning liberty scores to terminal nodes
- Cooling by 2

Example analyses of capturing races

We extend our methodology to be applied to more complex capturing races in which three or more groups are involved
Capturing Races (or *Semeais*)

A particular kind of life and death problem in which two adjacent confronting groups are fighting to capture the opponent’s group each other.
Essential block
- A block of black or white stones which must be saved from capture.
- Capturing an essential block immediately decides a semeai.

Liberty region
- A region which is surrounded by at least one essential block and some other essential blocks and safe blocks.
- A liberty region is called *external region* if its boundary does not consist of the essential blocks of different color.
Terminology

- **External liberty**: A liberty of an essential block in an external liberty region.

- **Shared liberty**: A common liberty of a Black’s essential block and a White’s essential block.

- **Eye liberty**: A liberty in a *nakade* shape eye.
In order to analyze semeais,

1. count the number of liberties for each subregion of an essential block,
2. sum up the numbers, then
3. the player who has the more liberties is the winner (if there is no shared liberty)

It’s a really simple procedure, but

the number of liberties may not always be a number, but a game whose value changes by each player’s move.

We count the number of liberties using CGT.
Liberty Counting Game

- Liberty Counting Game (LCG)
  - Score is the number of liberties of essential blocks
  - Black’s liberties are positive and White’s are negative

\[ \{ G^L | G^R \} \]

- Examples:

- It seems easy to get these CGT descriptions, but in fact, we have to resolve a subtle problem.
Semeai Games

- A semeai game is a sum of liberty counting games.
- Assumed that each summand has just one essential block.
- Rules for play are same as Go. Suicidal moves are forbidden except for the last winning move.
- Player who fills all the liberties of opponent’s all essential blocks in all summands is the winner.

In case of semeais, the smallest incentive is 1-ish, because the attacker can always fill the opponent’s liberties one by one.

So, *cooling by 2 degrees* works to analyze semeai games.
Supposed that $G$ is a semeai game and $g$ is $\text{Cool}(G, 2)$.

- **Case 1:** $g$ is an integer
  - If $g > 0$, Black wins.
  - If $g < 0$, White wins.
  - If $g = 0$, first player wins.

- **Case 2:** $n + 1 > g > n$ (for some integer $n$)
  - If Black plays first, he can round $g$ up to $n + 1$.
  - If White plays first, he can round $g$ down to $n$.
  - Check the resulting adjustment value using the conditions of case 1.

- **Case 3:** $g <> n$ (for some integer $n$)
  - If Black plays first, he can round $g$ up to $n + 1$.
  - If White plays first, he can round $g$ down to $n - 1$.
  - Check the resulting adjustment value using the conditions of case 1.
Example Problems

- Problem 1
- Problem 2
- Problem 3
- Problem 4
Complex capturing races are:

- capturing races which involve multiple essential blocks
Related Work

- Static analysis for complex capturing races

- Relation among groups of Black’s and White’s blocks are described using *semeai graph*.

- A method for static analysis to find the outcome of the entire capturing races using each local outcome between confronted groups.
Semeai Graph

- **Node**: Block of connected stones
  - Circled nodes are essential blocks
  - Values with nodes are eye liberties
  - Values of alive blocks are $\infty$

- **Link**
  - denotes that two blocks of different color come in contact, or share a region of empty points
  - Values with links are liberties between blocks
Outcome of Simple One-on-One Semeais

- Outcome of capturing races is determined by the values of $d_B, d_W, D, B$, and $W$

  - $d_B$: Total external liberties of Black’s essential blocks
  - $d_W$: Total external liberties of White’s essential blocks
  - $D$: Shared liberties
  - $B$: Eye liberties of Black’s essential blocks
  - $W$: Eye liberties of White’s essential blocks
Semeai Graph of Complex Semeais
For each pair of facing essential blocks,
assume that the other essential blocks are safe,
apply the procedure to decide the local outcome of the pair.
For each pair of facing essential blocks,
assume that the other essential blocks are safe,
apply the procedure to decide the local outcome of the pair.
For each pair of facing essential blocks,
assume that the other essential blocks are safe,
apply the procedure to decide the local outcome of the pair.
Outcome Table of One-vs-Two Semeais

(1) \( X_1 \leftrightarrow Y \rightarrow X_2 : Y \) wins
(2) \( X_1 \rightarrow Y \leftrightarrow X_2 : X \) wins (with some exceptions)
(3) \( X_1 \rightarrow Y \rightarrow X_2 : Y \) wins
(4) \( X_1 \leftrightarrow Y \leftrightarrow X_2 : Y \) wins
(5) \( X_1 \leftrightarrow Y \rightarrow X_2 : Y \) wins
(6) \( X_1 \leftrightarrow Y \leftrightarrow X_2 : Y \) wins
(7) \( X_1 \leftrightarrow Y \leftrightarrow X_2 : \) First player wins (with some exceptions)
(8) \( X_1 \leftrightarrow Y \rightarrow X_2 : Y \) wins
(9) \( X_1 \leftrightarrow Y \leftrightarrow X_2 : \) First player wins
(10) \( X_1 \leftrightarrow Y \leftrightarrow X_2 : Y \) wins

\[
\begin{align*}
X & \rightarrow Y : X \text{ wins if } X \text{ plays first and } Y \text{ can not win even if } Y \text{ plays first} \\
X & \leftrightarrow Y : \text{ First player wins} \\
X & \leftrightarrow Y : \text{ Seki}
\end{align*}
\]
Outcome Table of One-vs-Two Semeais

(1) \(X_1 \leftrightarrow Y \rightarrow X_2: \ Y \text{ wins}

(2) \(X_1 \rightarrow Y \leftrightarrow X_2: \ X \text{ wins} \) (with some exceptions)

(3) \(X_1 \rightarrow Y \rightarrow X_2: \ Y \text{ wins}

(4) \(X_1 \leftrightarrow Y \leftrightarrow X_2: \ Y \text{ wins}

(5) \(X_1 \leftrightarrow Y \rightarrow X_2: \ Y \text{ wins}

(6) \(X_1 \leftrightarrow Y \leftrightarrow X_2: \ Y \text{ wins}

(7) \(X_1 \leftrightarrow Y \leftrightarrow X_2: \ \text{First player wins} \) (with some exceptions)

(8) \(X_1 \leftrightarrow Y \rightarrow X_2: \ Y \text{ wins}

(9) \(X_1 \leftrightarrow Y \leftrightarrow X_2: \ \text{First player wins}

(10) \(X_1 \leftrightarrow Y \leftrightarrow X_2: \ Y \text{ wins}

\[
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X \leftrightarrow Y &: \ \text{Seki}
\end{align*}
\]
Example of One-vs-Two Semeais
Analysis of Complex Capturing Races using CGT

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Analyzing Complex Go Capturing Races

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Analysis of Complex Capturing Races using CGT

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CGTW2011 2011/01/12 19 / 37
Analysis of Complex Capturing Races using CGT

game: $\{0 \mid -4\}$

cooled value: $-2^*$
game: \( \{ 0 \mid -2 \mid -6 \} \)

cooled value: \(-2\uparrow\)
game: \{6 \mid 2\}

cooled value: 4*
game: 5
cooled value: 5
Analysis of Complex Capturing Races using CGT

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Analysis of Complex Capturing Races using CGT

$W_0$ vs $B_1 : -1\uparrow$
Analysis of Complex Capturing Races using CGT

$W_0$ vs $B_1 : -1^\uparrow$

$W_0$ vs $B_2 : \uparrow^*$
$W_0$ vs $B_1$ : $-1^\uparrow$

$W_0$ vs $B_2$ : $^\ast$

$B_1 \leftrightarrow W_0 \leftrightarrow B_2$

First player wins(?)
$W_0 \text{ vs } B_1 : -1 \uparrow$

Black’s winning move is $a$

$W_0 \text{ vs } B_2 : \uparrow^*$

Black’s winning moves are $b$ and $c$

They are different and Black cannot play both at the same time $\Rightarrow$ Black cannot win!
How to judge the total outcome using local outcomes

1. Construct a semeai graph from a complex capturing race as follows:
   - decompose essential blocks to subgames
   - calculate cooled value of liberty count for each subgame and assign the value to each link

2. analyze outcome between two local pair of nodes and get a set of winning moves

3. judge the total outcome using the determination table
   - check if two sets of winning moves have non empty intersection
   - if only one player has some common winning moves, the player wins, and
   - if both players have some common winning moves, the first player wins
Another Example

A set of Black's winning moves for $B_1 \leftrightarrow W_0$ is \{a\}

A set of Black's winning moves for $W_0 \leftrightarrow B_2$ is \{a, b\}

Black $a$ belongs in the both sets $\implies$ Black can win if Black plays first

and also White can win if White plays first.

The final outcome is the first player wins.
We suppose that there is no shared liberty, and $D_i = 0$ for all $i$.

A local semeai game is $G_i = W + d_0 + B_i + d_i$.

1’s side (White) wins the entire semeai, if he can win at least one local semeai of $G_i$.

In order for $n$’s side (Black) to win the entire semeai, he has to win all the local semeais of $G_i$ simultaneously.

AND/OR combinatorial game?
Summary

- One-vs-Two semeais with combinatorial games of liberty count
  - Static judgement table [K. Nakamura (2005)]
  - Local outcomes + Set of winning moves

- General case of 1-vs-\( n \) semeais is a new type of combinatorial games?
  - The \( n \)'s side cannot win if there is no common winning move for all local semeais.
  - But if there are some common winning moves for all local semeais, . . . ?
Outcome Table of One-vs-Two Semeais

(1) $X_1 \leftrightarrow Y \rightarrow X_2 : \text{Y wins}$

(2) $X_1 \rightarrow Y \leftrightarrow X_2 : \text{X wins (with some exceptions)}$

(3) $X_1 \rightarrow Y \rightarrow X_2 : \text{Y wins}$

(4) $X_1 \leftrightarrow Y \leftrightarrow X_2 : \text{Y wins}$

(5) $X_1 \leftrightarrow Y \rightarrow X_2 : \text{Y wins}$

(6) $X_1 \leftrightarrow Y \leftrightarrow X_2 : \text{Y wins}$

(7) $X_1 \leftrightarrow Y \leftrightarrow X_2 : \text{First player wins (with some exceptions)}$

(8) $X_1 \leftrightarrow Y \rightarrow X_2 : \text{Y wins}$

(9) $X_1 \leftrightarrow Y \leftrightarrow X_2 : \text{First player wins}$

(10) $X_1 \leftrightarrow Y \leftrightarrow X_2 : \text{Y wins}$

\[ \text{Diagram with nodes and edges representing different states and moves.} \]
Problem 1
Problem 1
Problem 1

subgame A:

game: \(\{4 \mid 0\}\)

cooled value: 2*

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Problem 1

subgame B:

game: \{6 \parallel 4 \parallel 0\}
cooled value: 4↑
Problem 1

subgame C:

game: $-7$

cooled value: $-7$
Problem 1

Total: \(-1^{\uparrow*}\)

\((= 2^{*} + 4^{\uparrow} - 7)\)

\(-1^{\uparrow*} \Leftrightarrow -1\)
Problem 1

Total: \(-1\uparrow\star\)

\((= 2\star + 4\uparrow - 7)\)

\(-1\uparrow\star << -1\)

Black: \(-1\uparrow\star \Rightarrow 0\)
Problem 1

Total: $-1^\uparrow\ast$

$(= 2^\ast + 4^\uparrow - 7)$

$-1^\uparrow\ast \leftrightarrow -1$

Black: $-1^\uparrow\ast \Rightarrow 0$

White: $-1^\uparrow\ast \Rightarrow -2$
Problem 1

Total: $-1^\uparrow*$

($= 2^\ast + 4^\uparrow - 7$)

$-1^\uparrow* \iff -1$

Black: $-1^\uparrow* \Rightarrow 0$

White: $-1^\uparrow* \Rightarrow -2$

The first player wins
Problem 1

- **Success for Black**
  - Black gets *tedomari*

- **Failure for Black**
  - White gets *tedomari*
Problem 2

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Problem 2

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CGTW2011 2011/01/12 31 / 37
Problem 2

subgame A:
  game:  \{4 \mid 0\}
  cooled value:  2^*

subgame B:
  game:  \{6 \parallel 4 \mid 0\}
  cooled value:  4↑
subgame A:
  game: \{4 \mid 0\}
  cooled value: \(2\)

subgame B:
  game: \{6 \parallel 4 \mid 0\}
  cooled value: \(4\)

subgame C:
  game: \{-5 \mid -9\}
  cooled value: \(-7\)
Problem 2

Total: $-1^\uparrow > -1$

subgame A:
- game: $\{4 \mid 0\}$
- cooled value: 2*

subgame B:
- game: $\{6 \mid 4 \mid 0\}$
- cooled value: 4↑

subgame C:
- game: $\{-5 \mid -9\}$
- cooled value: -7*
Problem 2

Total:  $-1\uparrow > -1$
Black:  $-1\uparrow \Rightarrow 0$

subgame A:
  game:  $\{4 \mid 0\}$
  cooled value:  $2^*$

subgame B:
  game:  $\{6 \parallel 4 \mid 0\}$
  cooled value:  $4\uparrow$

subgame C:
  game:  $\{-5 \mid -9\}$
  cooled value:  $-7^*$
Problem 2

subgame A:
  game:  {4 | 0}
  cooled value:  2

subgame B:
  game:  {6 || 4 | 0}
  cooled value:  4

subgame C:
  game:  {-5 | -9}
  cooled value:  -7

Total:  -1
Black:  -1  \Rightarrow  0
White:  -1  \Rightarrow  -1
The first player wins

subgame A:
  game: \{4 | 0\}
  cooled value: 2*

subgame B:
  game: \{6 || 4 | 0\}
  cooled value: 4↑

subgame C:
  game: \{-5 | -9\}
  cooled value: -7*

Total: \(-1↑ > -1\)
Black: \(-1↑ \Rightarrow 0\)
White: \(-1↑ \Rightarrow -1\)
Problem 2

- **Success for Black**
  
  \( (a \text{ and } b \text{ are miai}) \)
  
  Black gets *tedomari*

- **Failure for Black**
  
  \( (a \text{ and } c \text{ are miai}) \)
  
  White gets *tedomari*
Problem 3
Problem 3

subgames
Total = (2*) + (-2*) + (3*) + (2↓) + (-4→) + (6↑*) + 2 - 10 = -1↑
Range: 0 > -1↑ > -1

Black wins by one, if he plays first.
Total = (2*) + (-2*) + (3*) + (2↓) + (-4→) + (6↑*) + 2 - 10 = -1↑

Range: 0 > -1→ > -1

Black wins by one, if he plays first.
Problem 4

Black to move
Subgames

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<tr>
<th></th>
<th>value</th>
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<tbody>
<tr>
<td>A</td>
<td>$1\frac{3}{4}$</td>
</tr>
<tr>
<td>B</td>
<td>$2{0^3 \mid -3}$</td>
</tr>
<tr>
<td>C</td>
<td>$3{-1 \mid 0}$</td>
</tr>
<tr>
<td>D</td>
<td>$3\uparrow^*$</td>
</tr>
<tr>
<td>E</td>
<td>$1\frac{3}{4}$</td>
</tr>
<tr>
<td>F</td>
<td>$3$</td>
</tr>
<tr>
<td>G</td>
<td>$-1$</td>
</tr>
<tr>
<td>H</td>
<td>$-2\frac{1}{2}$</td>
</tr>
<tr>
<td>I</td>
<td>$-4\downarrow$</td>
</tr>
<tr>
<td>J</td>
<td>$-2^*$</td>
</tr>
<tr>
<td>K</td>
<td>$-1$</td>
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<tr>
<td>L</td>
<td>$-3{0 \mid \uplus^2}$</td>
</tr>
<tr>
<td>M</td>
<td>$-2{0^2 \mid \uplus^3}$</td>
</tr>
<tr>
<td>Total</td>
<td>$-1$ ish</td>
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