A Method for Analyzing Complex Go Capturing Races

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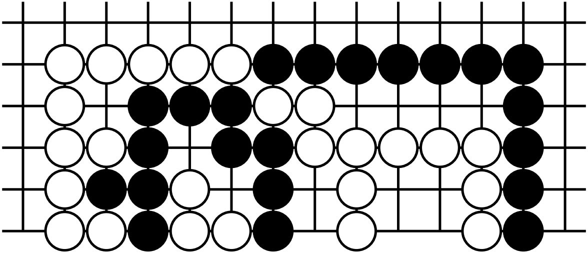
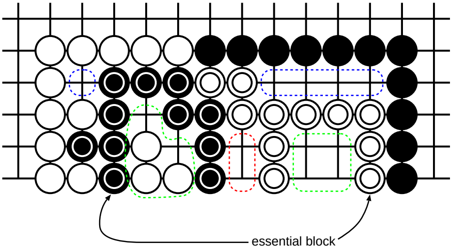
Capturing Races

- Capturing Races (or *Semeais*)
  - particular kind of life and death problem in which two adjacent confronting groups are fighting to capture the opponent's group each other

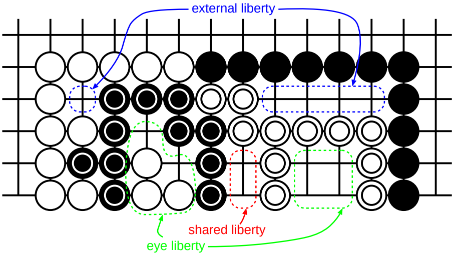


Terminology



- Essential block
  - A block of black or white stones which must be saved from capture.
  - Capturing an essential block immediately decides a semeai.
- Liberty region
  - A region which is surrounded by at least one essential block and some other essential blocks and safe blocks.
  - A liberty region is called external region if its boundary does not consist of the essential blocks of different color.

Terminology



- External liberty
  - A liberty of an essential block in an external liberty region
- Shared liberty
  - A common liberty of a Black's essential block and a White's essential block
- Eye liberty
  - A liberty in a *nakade* shape eye

Analysing Capturing Races

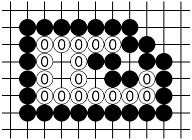
- In order to analyze semeais,
  1. count the number of liberties for each subregion of an essential block,
  2. sum up the numbers, then
  3. the player who has the more liberties is the winner (if there is no shared liberty)
- It's a really simple procedure, but
- the number of liberties may not always be a number, but a game whose value changes by each player's move.
- We count the number of liberties using CGT.

Liberty Counting Game

- Liberty Counting Game (LCG)
  - Score is the number of liberties of essential blocks
  - Black's liberties are positive and White's are negative

$$\left\{G^L \mid G^R\right\}$$

  - Examples:



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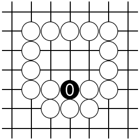
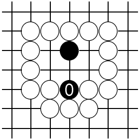


$\{3 \mid 0\}$



$\{4 \mid 0\}$

  - It seems easy to get these CGT descritions, but in fact, we have to resolve a subtle problem.

Semeai Games

- A semeai game is a sum of liberty counting games.
- Assumed that each summand has just one essential block.
- Rules for play are same as Go. Suicidal moves are forbidden except for the last winning move.
- Player who fills all the liberties of opponent's all essential blocks in all summands is the winner.

- In case of semeais, the smallest incentive is 1-ish, because the attacker can always fill the opponent's liberties one by one.
- So, *cooling by 2 degrees* works to analyze semeai games.

Evaluation Method for Semeai Games

Supposed that G is a semeai game and g is $Cool(G, 2)$.

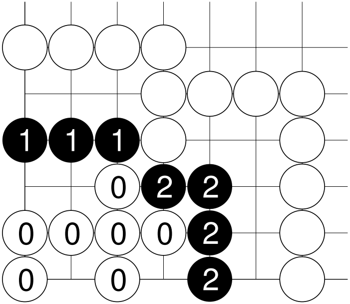
- Case 1: g is an integer
  - If $g > 0$, Black wins.
  - If $g < 0$, White wins.
  - If $g = 0$, first player wins.
- Case 2: $n + 1 > g > n$ (for some integer n)
  - If Black plays first, he can round g up to $n + 1$
  - If White plays first, he can round g down to n
  - Check the resulting adjustment value using the conditions of case 1.
- Case 3: $g <> n$ (for some integer n)
  - If Black plays first, he can round g up to $n + 1$
  - If White plays first, he can round g down to $n - 1$
  - Check the resulting adjustment value using the conditions of case 1.

Example Problems

- Problem 1 Go
- Problem 2 Go
- Problem 3 Go
- Problem 4 Go

Complex Capturing Races

- Complex capturing races are:
  - capturing races which involve multiple essential blocks

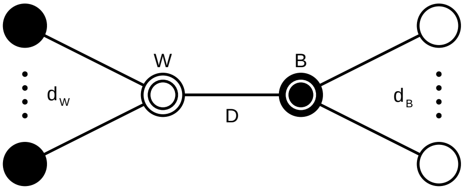


Related Work

- Static analysis for complex capturing races
  - Katsuhiko Nakamura: "Static analysis based on formal models and incremental computation in Go programming", Theoretical Computer Science, Vol.349, pp.184–201, (2005).
- Relation among groups of Black's and White's blocks are described using *semeai graph*.
- A method for static analysis to find the outcome of the entire capturing races using each local outcome between confronted groups

Semeai Graph

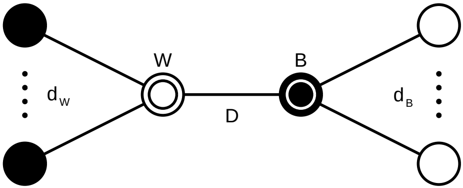
- Node: Block of connected stones
  - Circled nodes are essential blocks
  - Values with nodes are eye liberties
  - Values of alive blocks are ∞
- Link
  - denotes that two blocks of different color come in contact, or share a region of empty points
  - Values with links are liberties between blocks



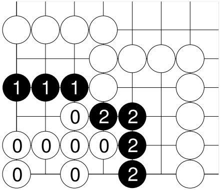
Outcome of Simple One-on-One Semeais

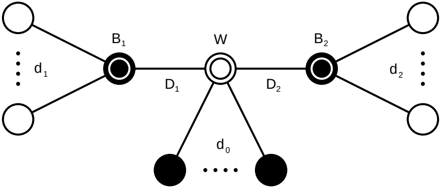
- Outcome of capturing races is determined by the values of d_B, d_W, D, B, and W

d_B	:	Total external liberties of Black's essential blocks
d_W	:	Total external liberties of White's essential blocks
D	:	Shared liberties
B	:	Eye liberties of Black's essential blocks
W	:	Eye liberties of White's essential blocks



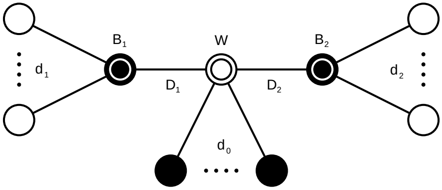
Semeai Graph of Complex Semeais





Process to Analyze Complex Semeais

- For each pair of facing essential blocks,
- assume that the other essential blocks are safe,
- apply the procedure to decide the local outcome of the pair



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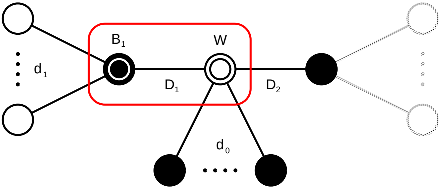

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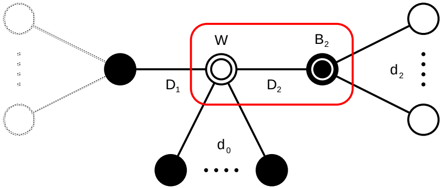


Outcome Table of One-vs-Two Semeais

(1) $X1 \Longleftarrow Y \Longrightarrow X2$: Y wins

(2) $X1 \Longrightarrow Y \Longleftarrow X2$: X wins (with some exceptions)

(3) $X1 \Longrightarrow Y \Longrightarrow X2$: Y wins

(4) $X1 \Longleftrightarrow Y \Longleftrightarrow X2$: Y wins

(5) $X1 \Longleftrightarrow Y \Longrightarrow X2$: Y wins

(6) $X1 \Longleftrightarrow Y \Longleftarrow X2$: Y wins

(7) $X1 \longleftrightarrow Y \longleftrightarrow X2$: First player wins (with some exceptions)

(8) $X1 \longleftrightarrow Y \Longrightarrow X2$: Y wins

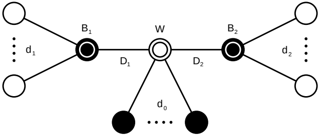
(9) $X1 \longleftrightarrow Y \Longleftarrow X2$: First player wins

(10) $X1 \longleftrightarrow Y \Longleftrightarrow X2$: Y wins

$X \Longrightarrow Y$: X wins if X plays first and Y can not win even if Y plays first

$X \longleftrightarrow Y$: First player wins

$X \Longleftrightarrow Y$: Seki



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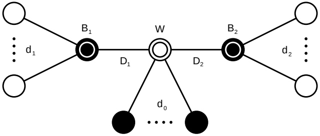
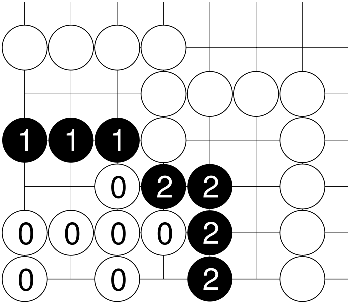
$X \Longrightarrow Y$: X wins if X plays first and Y can not win even if Y plays first

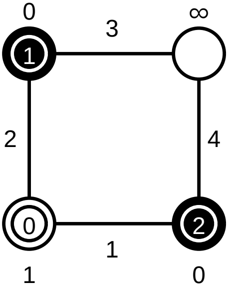
$X \longleftrightarrow Y$: First player wins

$X \Longleftrightarrow Y$: Seki

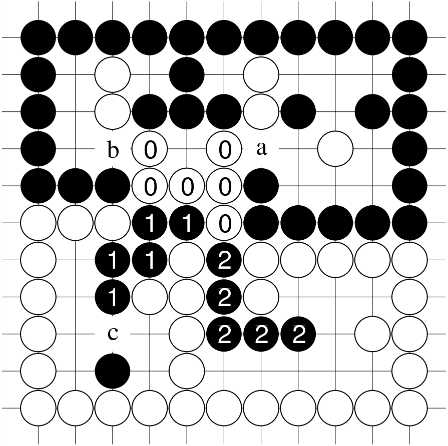


Example of One-vs-Two Semeais





Analysis of Complex Capturing Races using CGT



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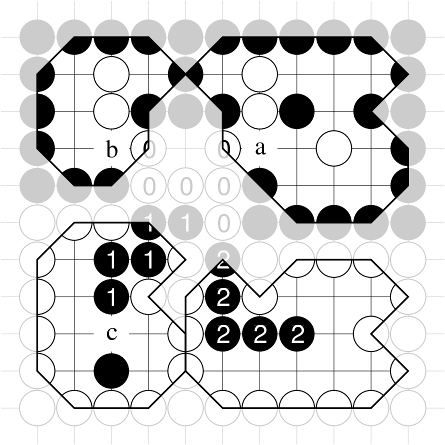
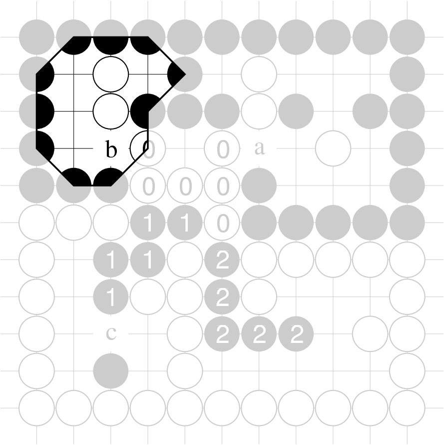
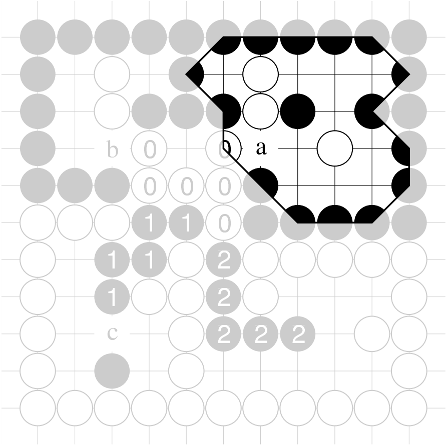




game: $\{0 \mid -4\}$

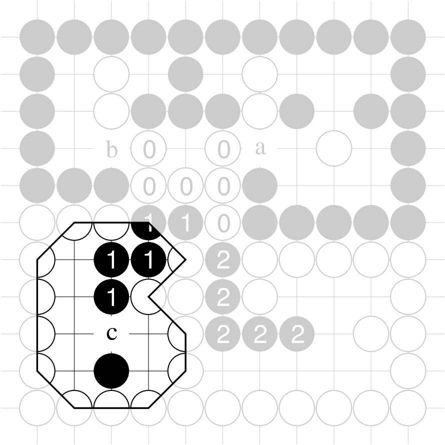
cooled value: $-2*$

Analysis of Complex Capturing Races using CGT



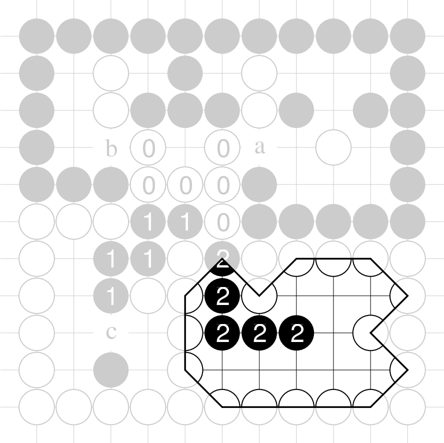
game: $\{0 \,||\, -2 \,|\, -6\}$

cooled value: $-2\uparrow$



game: $\{6 \mid 2\}$

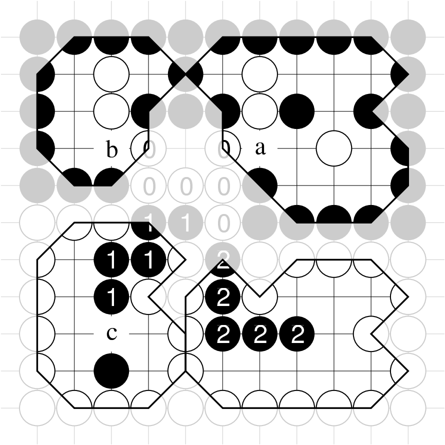
cooled value: $4*$

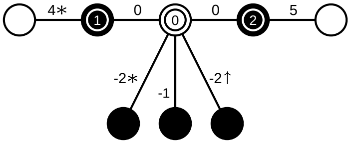


game: 5

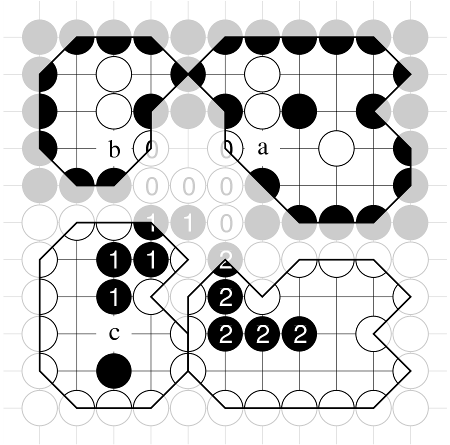
cooled value: 5

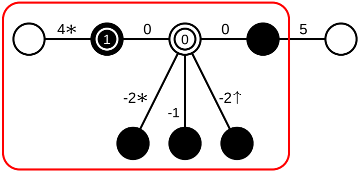
Analysis of Complex Capturing Races using CGT



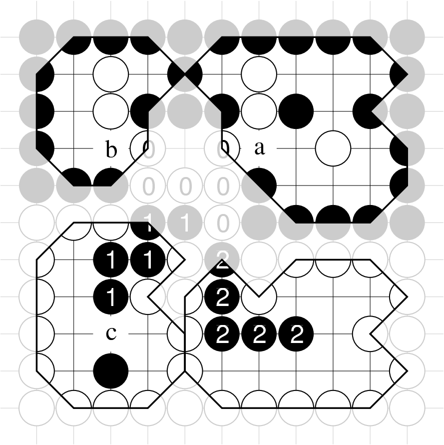


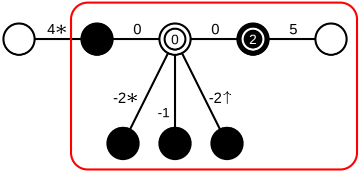
Analysis of Complex Capturing Races using CGT





W_0 vs B_1 : $-1\uparrow$

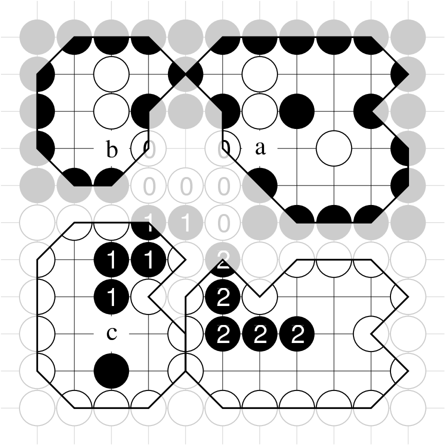


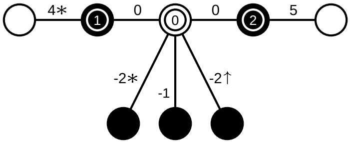


W_0 vs B_1 : $-1\uparrow$

W_0 vs B_2 : $\uparrow *$

Analysis of Complex Capturing Races using CGT





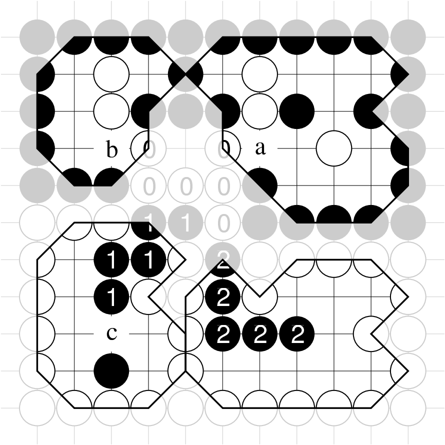
W_0 vs B_1 : $-1\uparrow$

W_0 vs B_2 : $\uparrow *$

$B_1 \longleftrightarrow W_0 \longleftrightarrow B_2$

Outcome Table

First player wins(?)



W_0 vs B_1 : $-1\uparrow$
Black's winning move is a

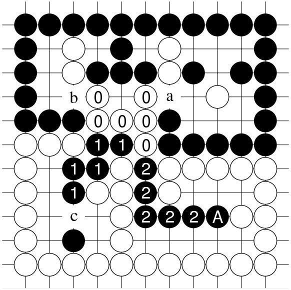
W_0 vs B_2 : $\uparrow *$
Black's winning moves are b and c

They are different and Black cannot play both at the same time $\Rightarrow$ Black cannot win!

How to judge the total outcome using local outcomes

1. Construct a semeai graph from a complex capturing race as follows:
   - decompose essential blocks to subgames
   - calculate cooled value of librety count for each subgame and assign the value to each link
2. analyze outcome between two local pair of nodes and get a set of winning moves
3. judge the total outcome using the determination table
   - check if two sets of winning moves have non empty intersection
   - if only one player has some common winning moves, the player wins, and
   - if both players have some common winning moves, the first player wins

Another Example



A is added in the lower right

- W_0 vs B_2 semeai

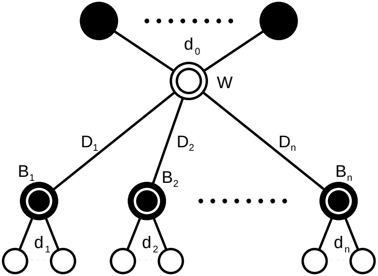
$$6 + (-2*) + (-2\uparrow) + (-1) = 1\uparrow*$$

- $1\uparrow* <> 1$ and the first player wins in this part
  - Black's winning moves are a and b
- Entire capturing race

$$B_1 \longleftrightarrow W_0 \longleftrightarrow B_2$$

- A set of Black's winning moves for $B_1 \longleftrightarrow W_0$ is $\{a\}$
- A set of Black's winning moves for $W_0 \longleftrightarrow B_2$ is $\{a, b\}$
- Black a belongs in the both sets $\Longrightarrow$ Black can win if Black plays first
- and also White can win if White plays first.
- The final outcome is the first player wins.

Semeai Graph of 1-vs-n Semeais



- We suppose that threre is no shared liberty, and $D_i = 0$ for all i.
- A local semeai game is $G_i = W + d_0 + B_i + d_i$.
- 1's side (White) wins the entire semeai, if he can win at least one local semeai of G_i.
- In order for n's side (Black) to win the entire semeai, he has to win all the local semeais of G_i simultaneously.
- AND/OR combinatorial game?

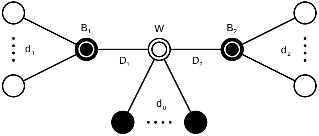
Summary

- One-vs-Two semeais with combinatorial games of liberty count
  - Static judgement table [K.Nakamura (2005)]
  - Local outcomes + Set of winning moves
- General case of 1-vs-n semeais is a new type of combinatorial games?
  - The n's side cannot win if there is no common winning move for all local semeais.
  - But if there are some common winning moves for all local semeais, . . . ?

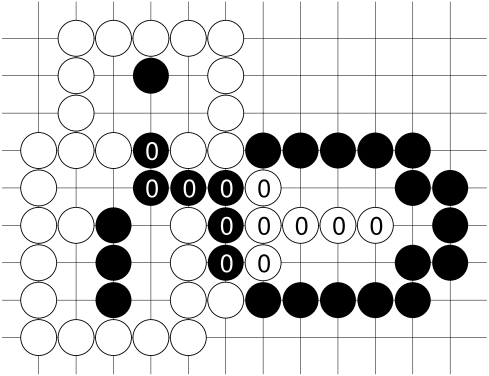
Outcome Table of One-vs-Two Semeais

(1) $X1 \Longleftarrow Y \Longrightarrow X2$: Y wins
(2) $X1 \Longrightarrow Y \Longleftarrow X2$: X wins (with some exceptions)
(3) $X1 \Longrightarrow Y \Longrightarrow X2$: Y wins
(4) $X1 \Longleftrightarrow Y \Longleftrightarrow X2$: Y wins
(5) $X1 \Longleftrightarrow Y \Longrightarrow X2$: Y wins
(6) $X1 \Longleftrightarrow Y \Longleftarrow X2$: Y wins
(7) $X1 \longleftrightarrow Y \longleftrightarrow X2$: First player wins (with some exceptions)
(8) $X1 \longleftrightarrow Y \Longrightarrow X2$: Y wins
(9) $X1 \longleftrightarrow Y \Longleftarrow X2$: First player wins
(10) $X1 \longleftrightarrow Y \Longleftrightarrow X2$: Y wins

return



Problem 1



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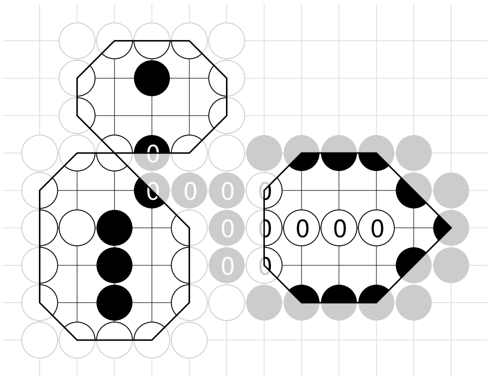

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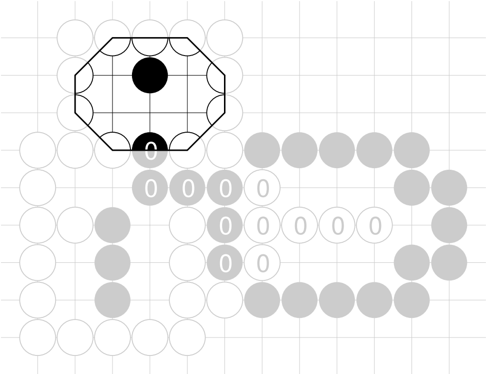
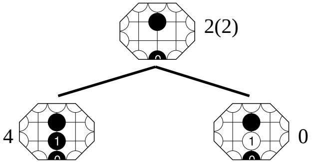


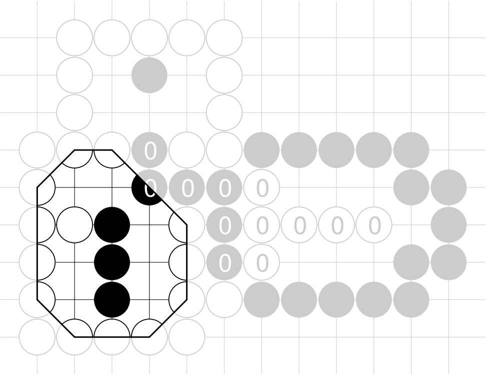
subgame A:



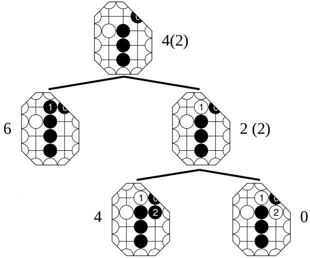
game: $\{4 \mid 0\}$

cooled value: $2*$

Problem 1



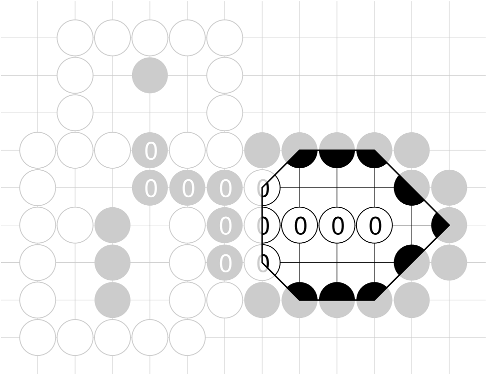
subgame B:



game: $\{6 \,||\, 4 \mid 0\}$

cooled value: $4\uparrow$

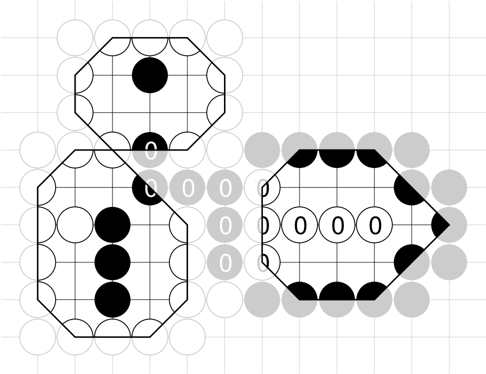
Problem 1



subgame C:

game: -7
cooled value: -7

Problem 1

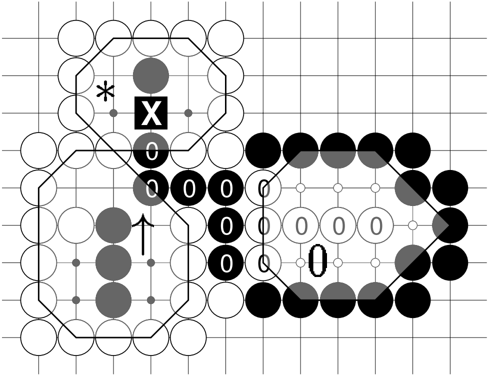


Total: $-1\uparrow *$

$(= \ 2* \ + \ 4\uparrow \ - \ 7)$

$-1\uparrow * \ <> \ -1$

Problem 1



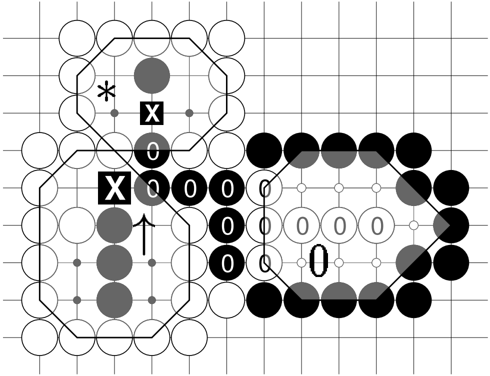
Total: $-1{\uparrow}*$

$(= \ 2* \ + \ 4{\uparrow} \ - \ 7)$

$-1{\uparrow}* \ <> \ -1$

Black: $-1{\uparrow}* \quad \Rightarrow 0$

Problem 1



Total: $-1{\uparrow}*$

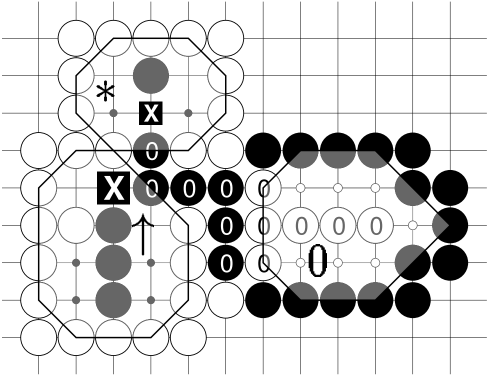
$(= 2* + 4{\uparrow} - 7)$

$-1{\uparrow}* <> -1$

Black: $-1{\uparrow}* \quad \Rightarrow 0$

White: $-1{\uparrow}* \quad \Rightarrow -2$

Problem 1



Total: $-1{\uparrow}*$

$(= 2* + 4{\uparrow} - 7)$

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Black: $-1{\uparrow}* \Rightarrow 0$

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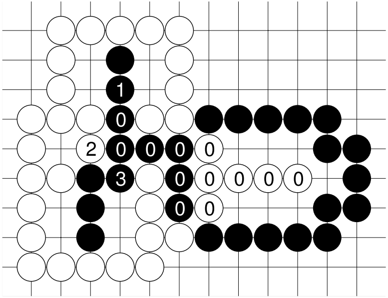
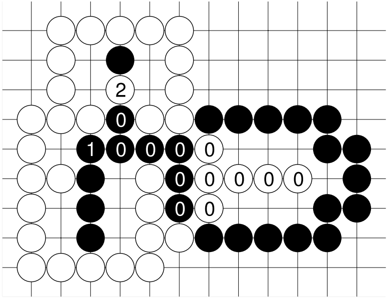
The first player wins

- Success for Black



Black gets *tedomari*

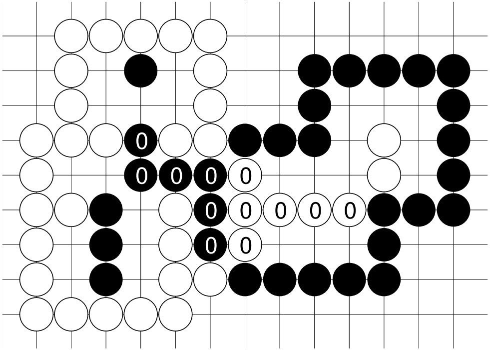
- Failure for Black



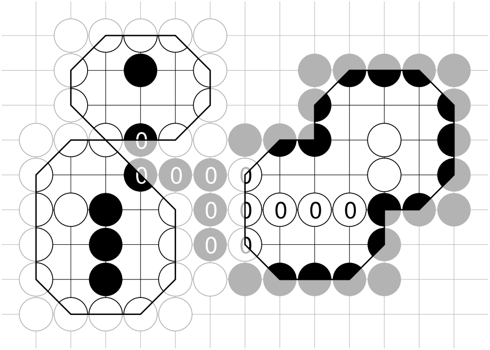
White gets *tedomari*

return

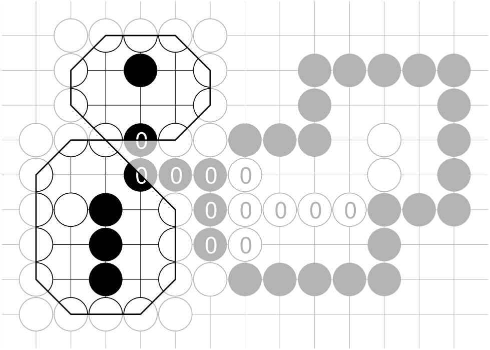
Problem 2



Problem 2



Problem 2



subgame A:

game : $\{4 \mid 0\}$

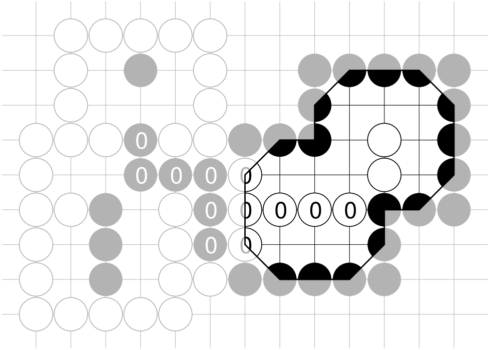
cooled value : $2*$

subgame B:

game: $\{6 \,||\, 4 \mid 0\}$

cooled value : $4\uparrow$

Problem 2

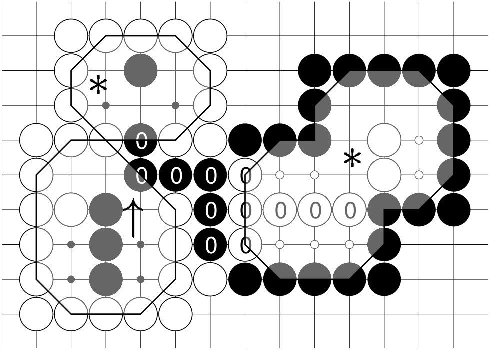


subgame A:
game : $\{4 \mid 0\}$
cooled value : $2*$

subgame B:
game: $\{6 \mid\mid 4 \mid 0\}$
cooled value : $4\uparrow$

subgame C:
game : $\{-5 \mid -9\}$
cooled value : $-7*$

Problem 2



Total: $-1\uparrow \quad > -1$

subgame A:

game : $\{4 \mid 0\}$

cooled value : $2*$

subgame B:

game: $\{6 \mid\mid 4 \mid 0\}$

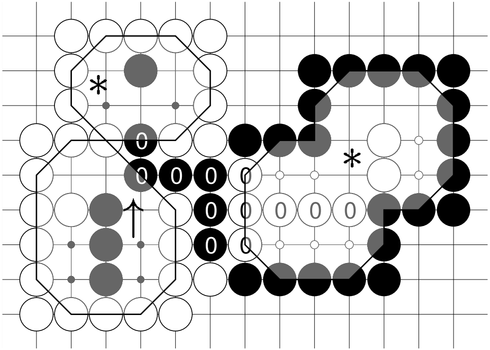
cooled value : $4\uparrow$

subgame C:

game : $\{-5 \mid -9\}$

cooled value : $-7*$

Problem 2



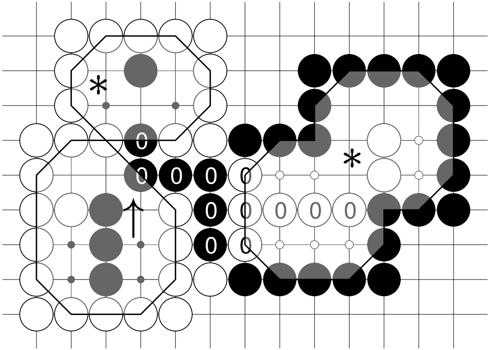
Total: $-1\uparrow \quad > -1$
Black: $-1\uparrow \quad \Rightarrow 0$

subgame A:
game : $\{4 \mid 0\}$
cooled value : $2*$

subgame B:
game: $\{6 \mid\mid 4 \mid 0\}$
cooled value : $4\uparrow$

subgame C:
game : $\{-5 \mid -9\}$
cooled value : $-7*$

Problem 2



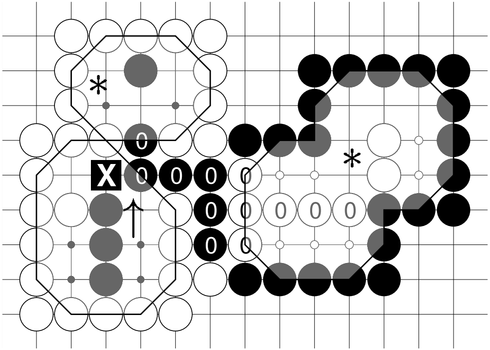
Total: $-1\uparrow \quad > -1$
Black: $-1\uparrow \quad \Rightarrow 0$
White: $-1\uparrow \quad \Rightarrow -1$

subgame A:
game : $\{4 \mid 0\}$
cooled value : $2*$

subgame B:
game: $\{6 \,||\, 4 \mid 0\}$
cooled value : $4\uparrow$

subgame C:
game : $\{-5 \mid -9\}$
cooled value : $-7*$

Problem 2



Total: $-1\uparrow \quad > \; -1$
Black: $-1\uparrow \quad \Rightarrow 0$
White: $-1\uparrow \quad \Rightarrow -1$

The first player wins

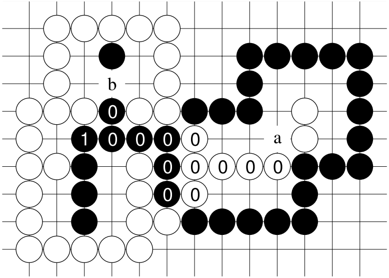
subgame A:
game : $\{4 \mid 0\}$
cooled value : $2*$

subgame B:
game: $\{6 \mid\mid 4 \mid 0\}$
cooled value : $4\uparrow$

subgame C:
game : $\{-5 \mid -9\}$
cooled value : $-7*$

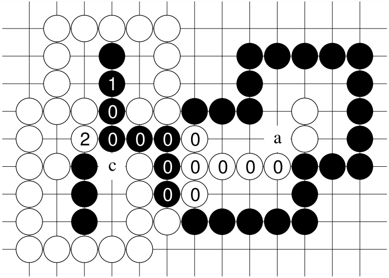
Problem 2

- Success for Black



(a and b are miai)
Black gets *tedomari*

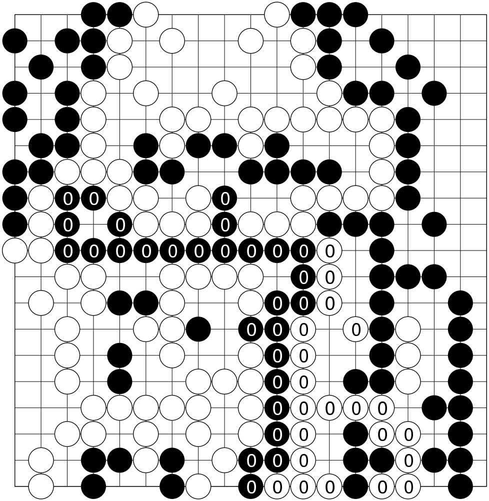
- Failure for Black



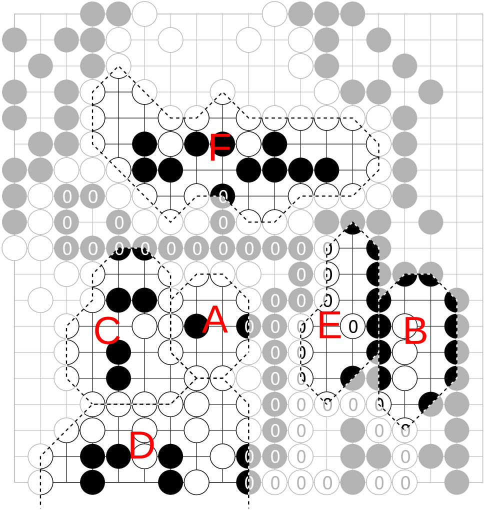
(a and c are miai)
White gets *tedomari*

return

Problem 3

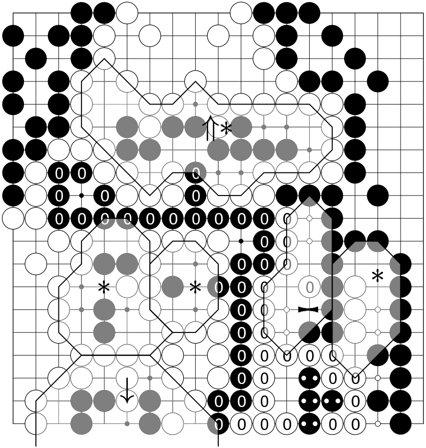


Problem 3



subgames

Analysis



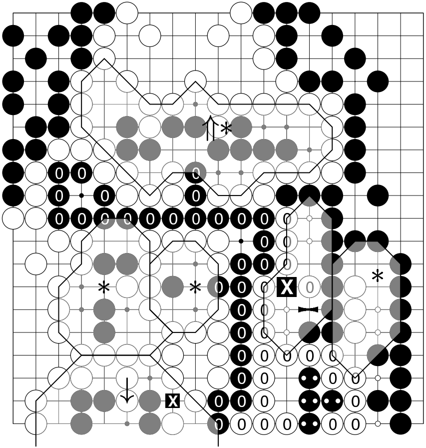
$$\text{Total} \;=\; (2\,*) + (-2\,*) + (3\,*) + (2\downarrow) + (-4\,\text{⧿}) + (6\Uparrow *) + 2 - 10 \;=\; -1\uparrow\text{⧿}$$

$$\text{Range}:\quad 0 > -1\uparrow\text{⧿} > -1$$

- Black wins by one, if he plays first.

return

Analysis



$$\text{Total} = (2*) + (-2*) + (3*) + (2\downarrow) + (-4\dashv) + (6\Uparrow*) + 2 - 10 = -1\uparrow\dashv$$

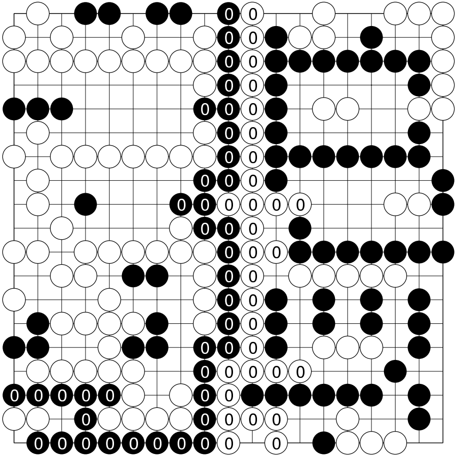
$$\text{Range}: \quad 0 > -1\uparrow\dashv > -1$$

- Black wins by one, if he plays first.

return

Problem 4

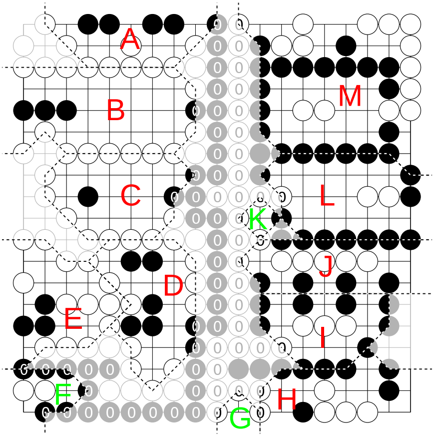
- Black to move



return

Problem 4

- Subgames



	value
A	$1\frac{3}{4}$
B	$2\{⧿3 \mid 0^3\}$
C	$3\{⧿1 \mid 0\}$
D	$3{\uparrow}*$
E	$1\frac{3}{4}$
F	3
G	-1
H	$-2\frac{1}{2}$
I	$-4{\downarrow}$
J	$-2*$
K	-1
L	$-3\{0 \mid ⧾2\}$
M	$-2\{0^2 \mid ⧾3\}$
Total	-1 ish

return